PSYC214: Statistics
Lecture 5 – Summary Part 1

Michaelmas Term
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Lecture 1 – Measurement, variance and inferential statistics

Agenda/Content

- Experimental science
- Variables
- Descriptive statistics
    - Levels of measurement
    - Measures of central tendency
    - Measures of variability
- Distributions
- Inferential statistics and hypotheses
- Within and between participant designs

Experimental science

Population versus sample

- Population is every individual you are interested in
- The sample is a subset of your population of interest. We examine samples because it is typically impossible to sample everyone in the population

Experimental science

Population versus sample

- You should always opt for **random sampling**, where you pick your sample randomly
- However, in reality, we often use opportunity sampling where we recruit who we have access to

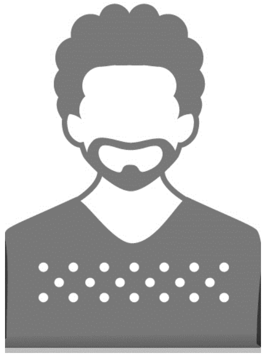
Variables

Independent Variable

- The variable (FACTOR) the experimenter manipulates or changes, which may be assumed to have a direct effect (i.e., influences change) on the dependent variable.

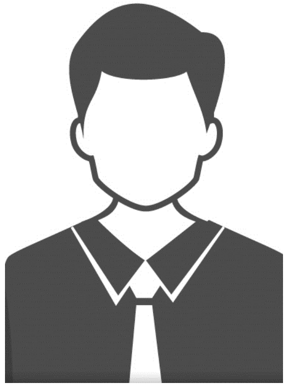
Dependent Variable

- The outcome of interest. It is the variable being tested and measured in an experiment. It is 'dependent' on the effect (i.e., influence) of the independent variable.



Statistics

- Use descriptive statistics to describe characteristics and tendencies of your sample
- Use inferential statistics to determine whether the performance and characteristics of your sample generalizes to the population

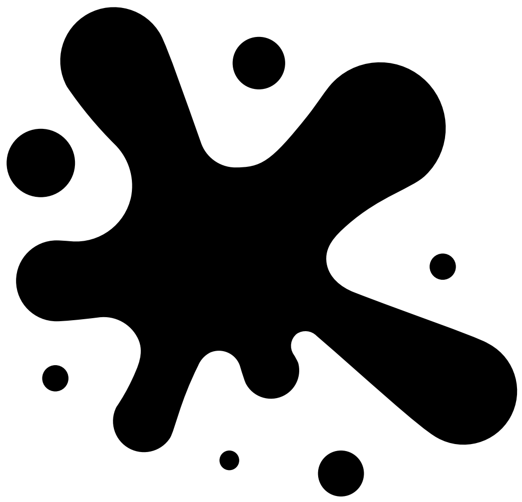


Descriptive statistics

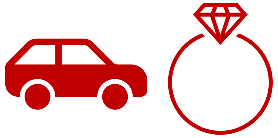
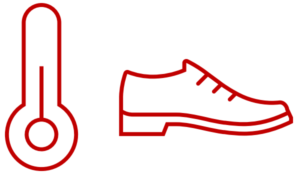
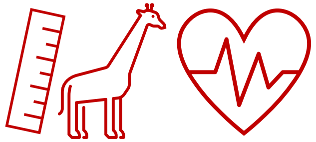
1. Levels of measurement
2. Measures of central tendency
3. Measures of variability

1. Levels of measurement

Nominal, Ordinal, Interval, Ratio



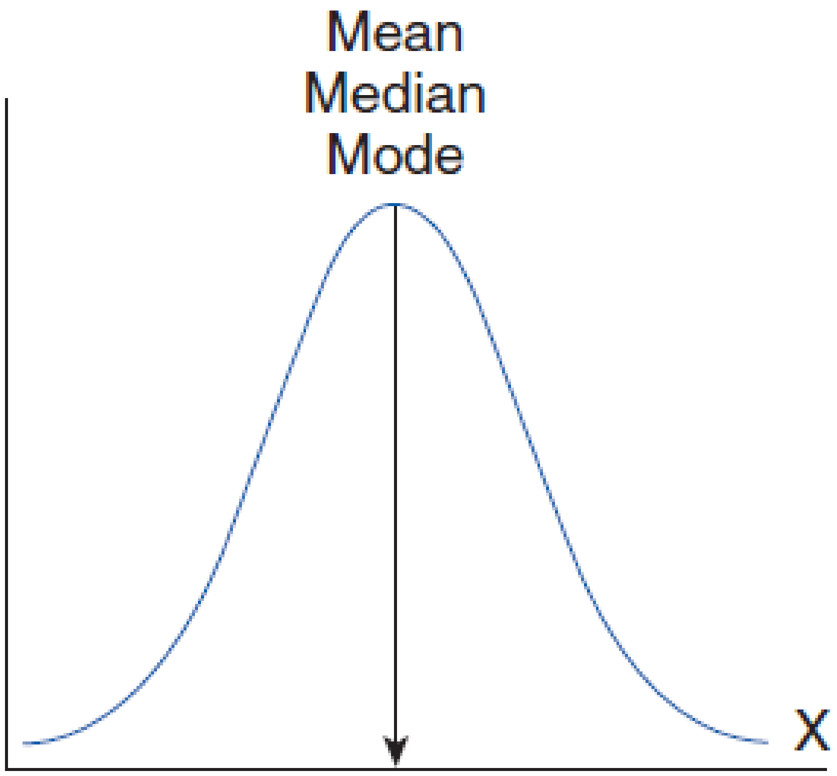
1. Levels of measurement - Examples

	Nominal	Ordinal	Interval	Ratio
Categories, Names				
Rank or order				
Known and proportionate intervals				
True zero				

2. Measures of central tendency

A single value that describes the way in which a group of data clusters around a central value, i.e., the centre of the data set

- There are three measures of central tendency
  - Mode
  - Median
  - Mean



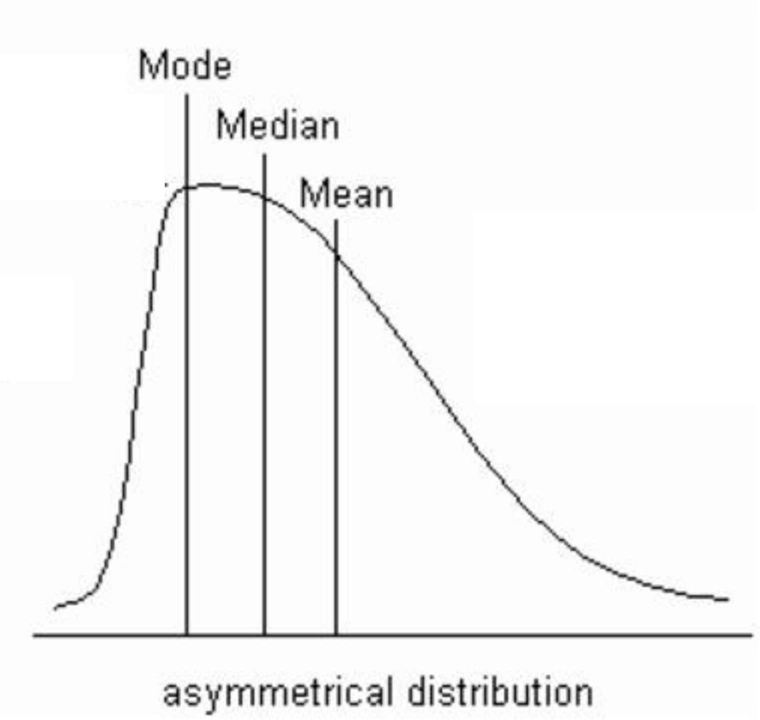
2. Measures of central tendency

	Nominal	Ordinal	Interval	Ratio
Categories, Names	Mode, % frequencies	Mode, % frequencies	Mode, % frequencies	Mode, % frequencies
Rank or order		Median, percentile	Median, percentile	Median, percentile
Known and proportionate intervals			Mean, standard deviation	Mean, standard deviation
True zero				All above

2. Measures of central tendency - Median

The middle number when data are ordered

- Level of measurement: Ordinal or interval/ratio
- Shape of distribution: Highly skewed



2. Measures of central tendency - Mean ($\bar{X}$)

The average, i.e., the sum (Σ) of all scores (x) divided by the number of scores (N)

Total set of scores

$$\bar{X} = \frac{\sum X}{N}$$

$$\bar{X} = \frac{5 + 7 + 7 + 6 + 2 + 3 + 4 + 5}{8}$$

Mean of a set of numbers

Number of scores

2. Measures of central tendency - Mean (X̄)

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The average, i.e., the sum (Σ) of all scores (x) divided by the number of scores (N)

Total set of scores

$$\bar{X} = \frac{\sum X}{N}$$

Mean of a set of numbers

Number of scores

$$\bar{X} = 4.875$$

3. Measures of variability

The spread or dispersion of scores in relation to the midpoint of data.

- Range
- Sum of squares
- Variance
- Standard deviation

3. Measures of variability - why care?

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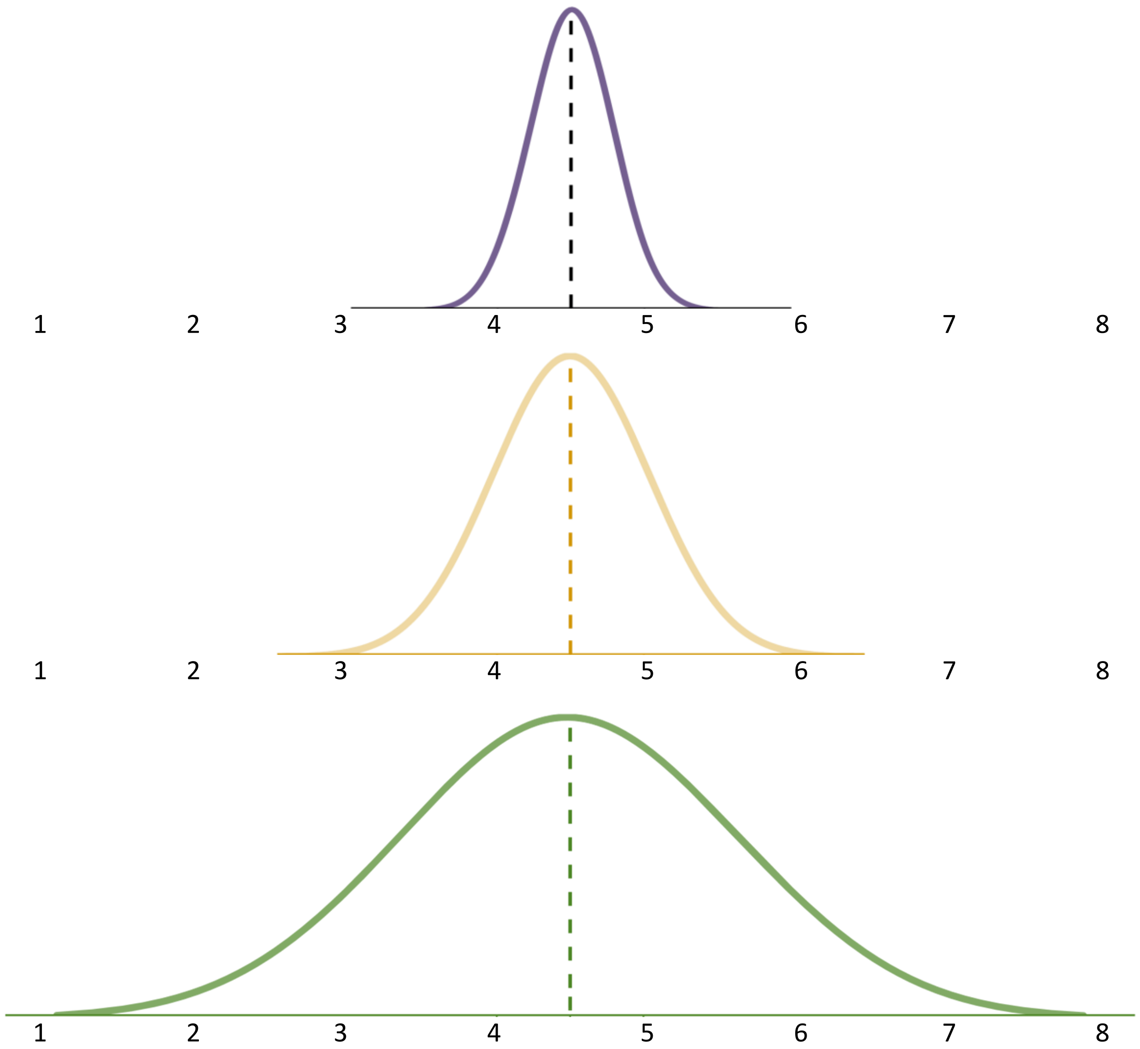
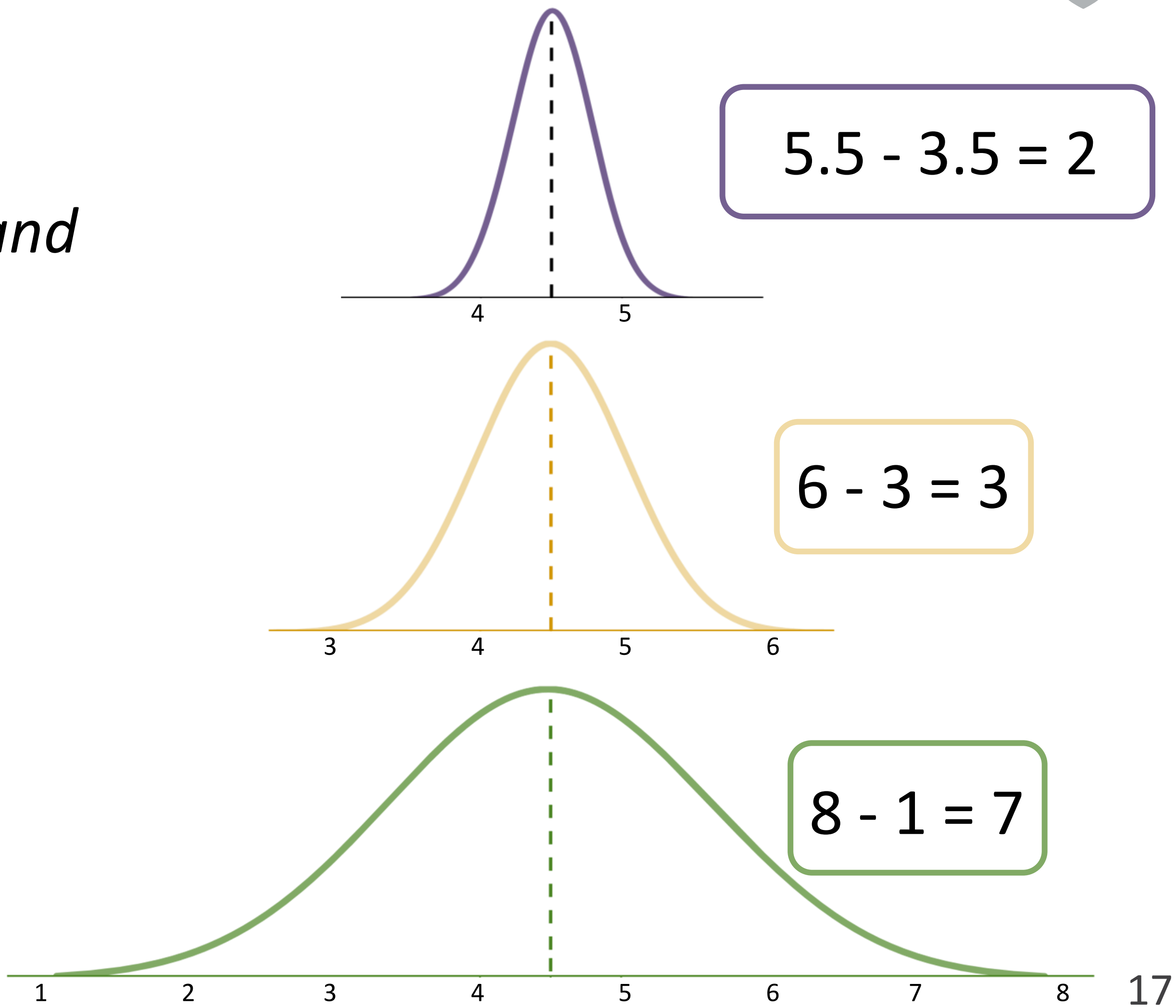


3. Measures of variability - range

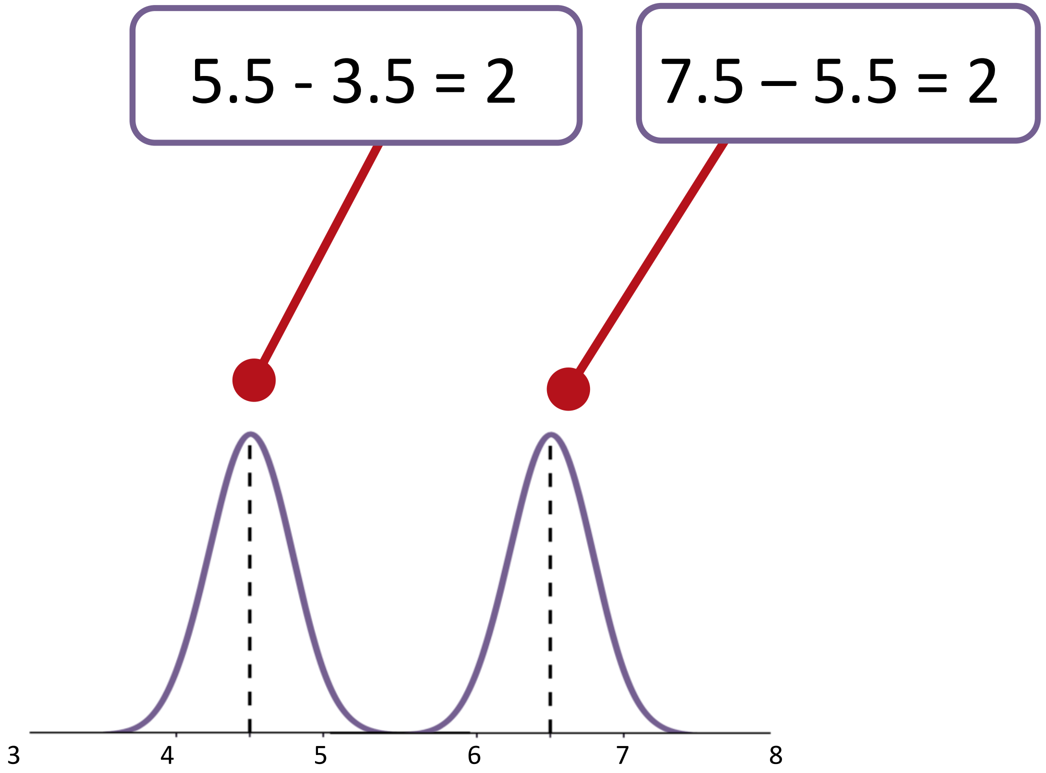
The difference between the highest and lowest score

- Subtract the lowest value in the distribution by the highest value



3. Measures of variability - range

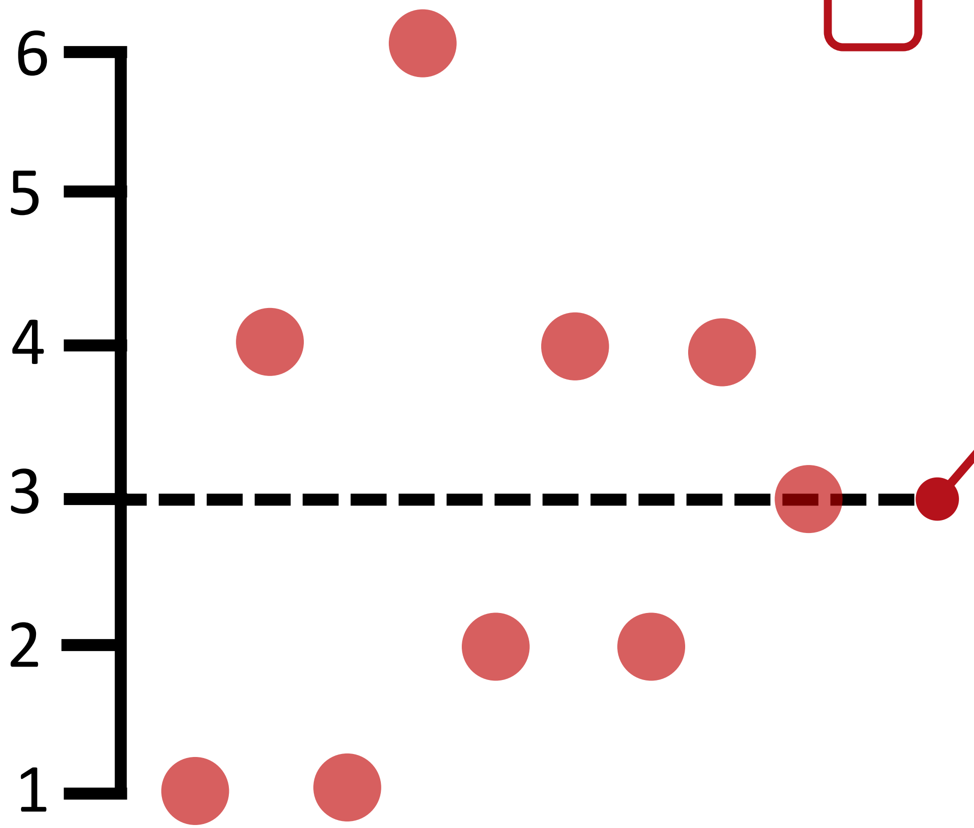
When is it not useful?



3. Measures of variability - sum of squares

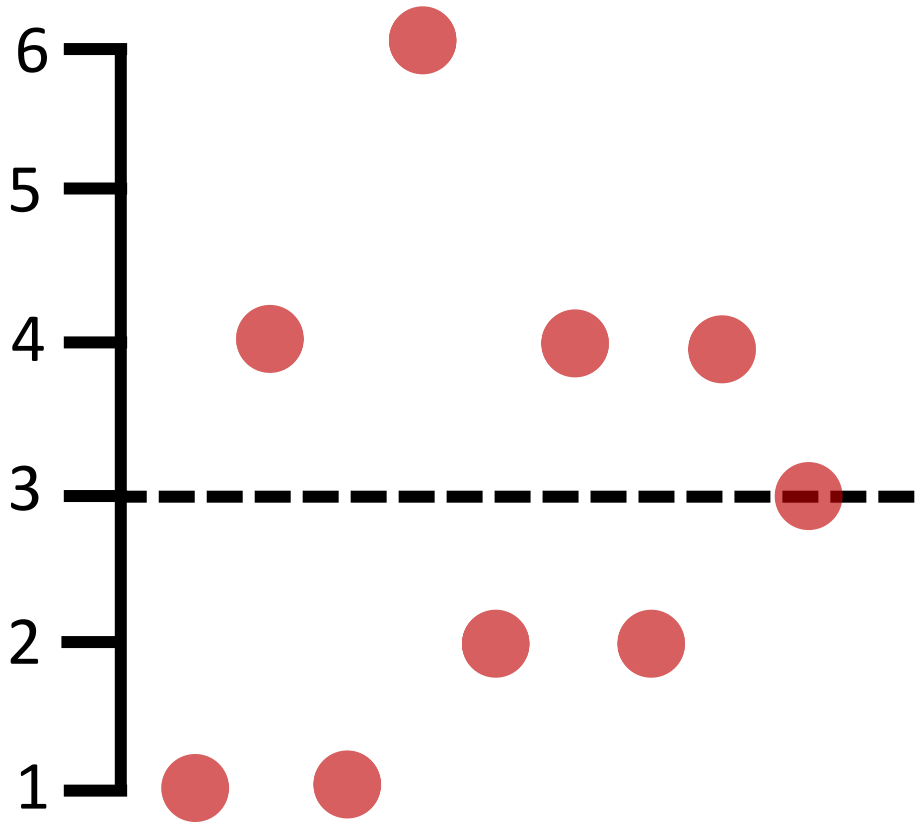
1. Calculate difference $X - \mu$

Population mean



3. Measures of variability - sum of squares

1. Calculate difference $X - \mu$



Data point	χ - μ
χ1	-2
χ2	1
χ3	-2
χ4	3
χ5	-1
χ6	1
χ7	-1
χ8	1
χ9	0
Total	0

3. Measures of variability - sum of squares

1. Calculate difference $X - \mu$
2. Calculate the sum of squares

Sum of squares (SS) $= \text{å}(m - x_i)^2$

å is the sum of all data

m is the population mean

x_i is each data point

Data point	χ - μ	(χ - μ)²
χ1	-2	4
χ2	1	1
χ3	-2	4
χ4	3	9
χ5	-1	1
χ6	1	1
χ7	-1	1
χ8	1	1
χ9	0	0
Total	0	22

3. Measures of variability - variance

- Variance: Average deviation around the mean of a distribution (average of sum of squares)

$$\text{Variance } (\sigma^2) = \frac{\sum (\mu - x_i)^2}{n - 1}$$

Sum of squares

Degrees of freedom

Where μ is the mean
x_i is each data point
n is the number of data points

3. Measures of variability – standard deviation

- Standard deviation (σ): Measure of the typical deviation from the mean. It is the squared root of the variance

$$\text{Standard Deviation } (\sigma) = \sqrt{\frac{\sum (\mu - x_i)^2}{n - 1}}$$

Variance

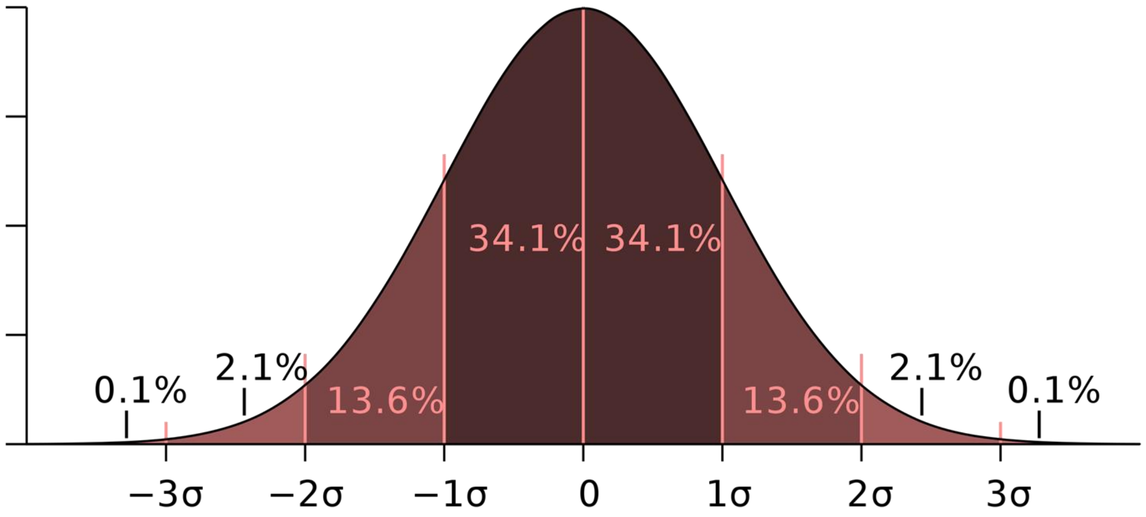
Where μ is the mean

x_i is each data point

n is the number of data points

3. Measures of variability – standard deviation

- Standard deviation (σ): Measure of the typical deviation from the mean. It is the squared root of the variance



Inferential statistics

1. Allow you to draw conclusions based on extrapolations
2. Use data from the sample of participants in the experiment to compare the treatment groups and make generalizations about the larger population of participants
3. Provide a quantitative method to decide if the null hypothesis (H_0) should be rejected

Inferential statistics - Hypotheses

H_0 the Null Hypothesis

- H_0: there is no significant difference between the conditions/groups and the null hypothesis is accepted.
- Under H_0, the samples come from the <u>same</u> population.

H_1 the Experimental Hypothesis

- H_1: there is a significant difference between the conditions/groups and the null hypothesis is rejected.
- Under H_1, the samples come from the <u>different</u> populations.

Inferential statistics - (Non)parametric tests

- Statistical tests can be separated into:
  - Parametric
  - Non-parametric

While parametric tests are the norm in psychology and are generally more powerful than non-parametric tests, they require that the scores be an interval or ratio measure and there needs to be homogeneity of variance


Savage Chickens
by Doug Savage
ACCORDING TO THIS CHART, THE ARROW IS MUCH HIGHER ON THE RIGHT HAND SIDE
BRILLIANT!
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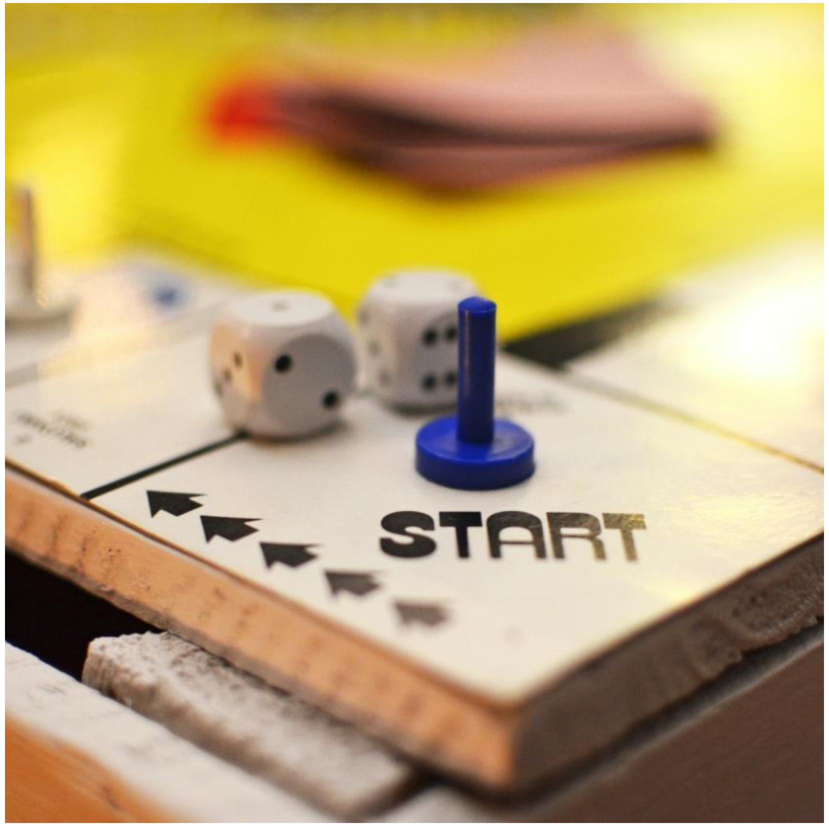
One factor between-participants ANOVA

Agenda/Content for Lecture 2

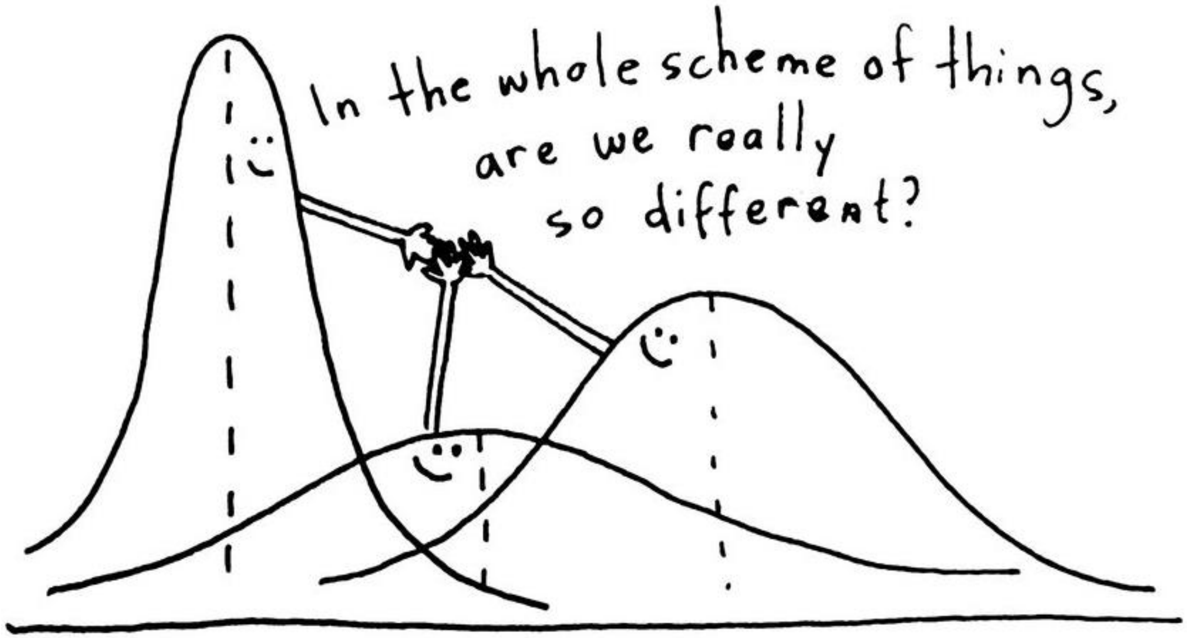
- Introduction to analysis of variance (ANOVA)
- Introduction to one factor between-participants design
- Sources of variability in data
- Calculating within-group and between-group variances
- Degrees of Freedom
- Producing the F-statistic



Introduction to analysis of variance

What do you need for a one factor between participants ANOVA?

- Three or more separate groups
- ONE categorical independent variable (i.e., one factor)
- One <u>continuous</u> dependent variable (outcome measure)



Source: Questionpro

Sources of variability in data

1. Treatment effects
2. Individual differences
3. Random (residual) errors

Within-group variability?

Between-group variability?

Treatment effects

- The effects of the independent variable
- This is what we want!
- We want people who are treated differently because of our intervention to behave differently

Individual differences

- Some individuals may be more proficient in memory recall
- Maybe some individuals have experience of similar tasks
- Some may have ignored instructions or had lower attention spans / motivation
- A control group can employ their own strategy, increasing the variability

Random (residual) errors

- Ideally a participant would have a 'true level' at which they perform, which can always be measured accurately

1. Varying external conditions – e.g., temperature, time of day
2. State of participant (e.g. tired?)
3. Experimenter's ability to measure accurately…

...Experimenter effects

- Experimenters need to minimise these, so not to obscure the treatment effect
- Spread data away from the true means – i.e., increase variability and standard errors
- Reduce confidence in our estimates and a randomly plucked sample

Within- and between- group variability

<u>Within-group variability</u>

The extent to which participants within a single group or population differ, despite receiving the same treatment

Within-group variability?

<u>Between-group variability</u>

The extent to which overall groups differ from one another (hopefully because of our treatment) * but also Individual differences!

Between-group variability?

Introduction to analysis of variance

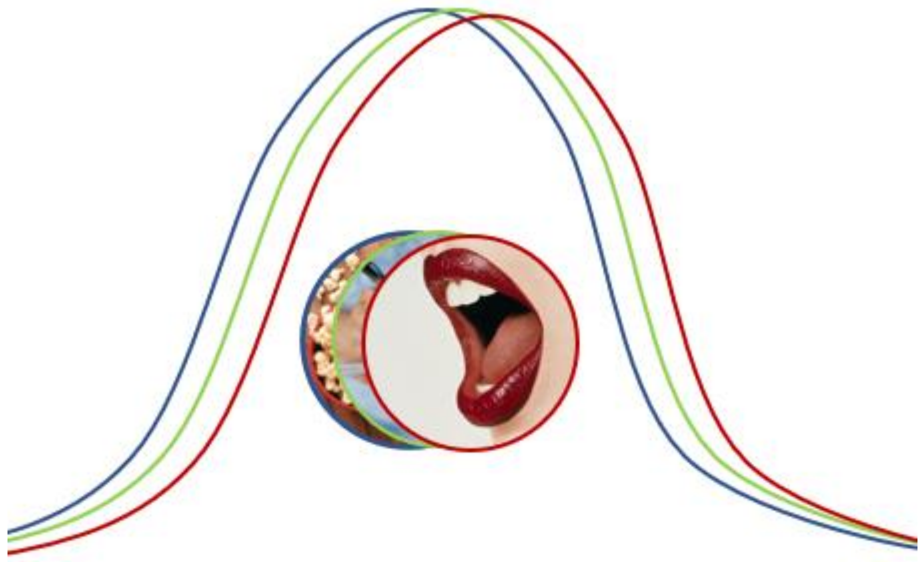
Factors and levels (Example 3)

- Factor: **experimental** condition
- 3 levels:
  - A_1 Verbal negative feedback
  - A_2 Written negative feedback
  - A_3 Control (no feedback)

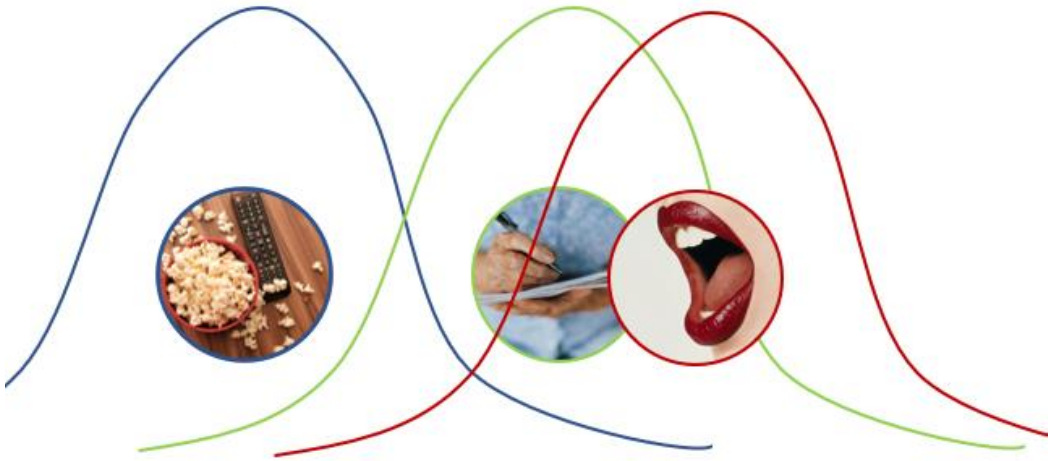
A_1 A_2 A_3

Testing for differences

- H_0 the Null Hypothesis
- Under H0, the samples come from the <u>same</u> population
- $\mu_1 = \mu_2 = \mu_3$ [No difference in the population means]
- Experimental effect = 0
- All differences are due to individual differences + random (residual) errors

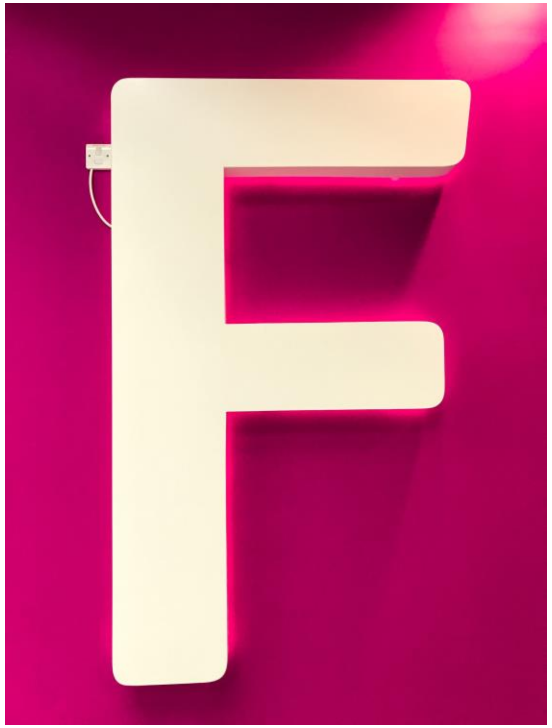


Testing for differences



- H1 the Experimental Hypothesis
- Under H1, the samples come from the different populations.
- $\mu_1 \neq \mu_2 \neq \mu_3$ [Population means are different]
- Experimental effect $\neq 0$
- Differences are due to individual differences, random (residual) errors AND the experimental effect

Introduction to analysis of variance



$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{Signal}}{\textbf{Noise}}$$

$$F = \frac{\textbf{Signal}}{\text{Noise}}$$

The F ratio

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{treatment effects + individual differences + random (residual) errors}}{\text{individual differences + random (residual) errors}}$$

$$F = \frac{\text{treatment effects + experimental error}}{\text{experimental error}}$$

Introduction to analysis of variance

$$F = \frac{\text{Signal}}{\text{Noise}}$$

$$F = \frac{\text{Signal}}{\text{Noise}}$$

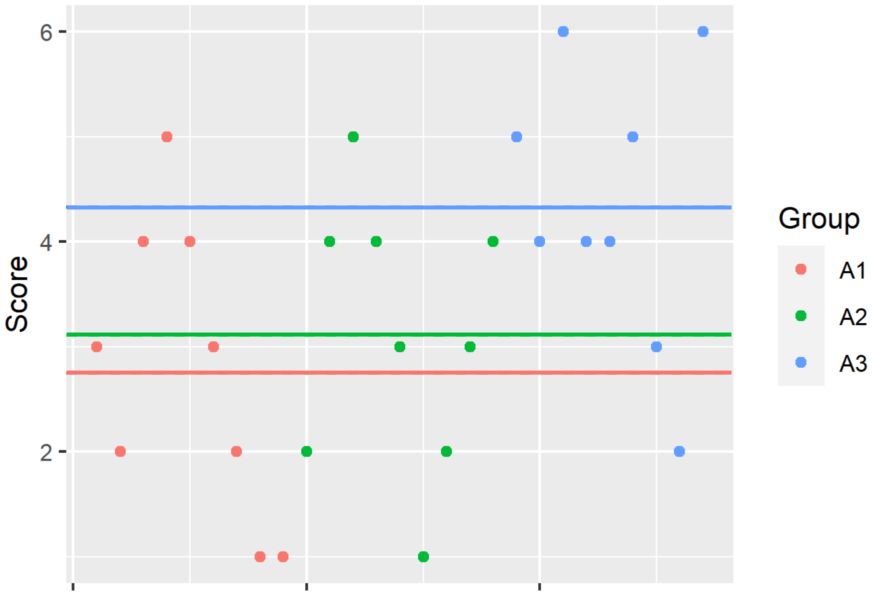
The more treatment effects are standing out away from experimental error – i.e., the larger the signal is from the noise, the larger in magnitude the F value.

The larger the F, the less likely that differences in scores are caused by chance.

Mean ($\bar{A}$)



A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1$ = 2.78	$\bar{A}_2$ = 3.11	$\bar{A}_3$ = 4.33



Grand Mean ($\bar{Y}$)

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A_1 scores	A_2 scores	A_3 scores
3	2	5
2	4	4
4	5	6
5	4	4
4	3	4
3	1	5
2	2	3
1	3	2
1	4	6
$\bar{A}_1$ = 2.78	$\bar{A}_2$ = 3.11	$\bar{A}_3$ = 4.33

$$\bar{Y} = \frac{\bar{A}_1 + \bar{A}_2 + \bar{A}_3 + \ \ldots \bar{A}_k}{k}$$

$\bar{Y} = The\ grand\ mean\ of\ averages$

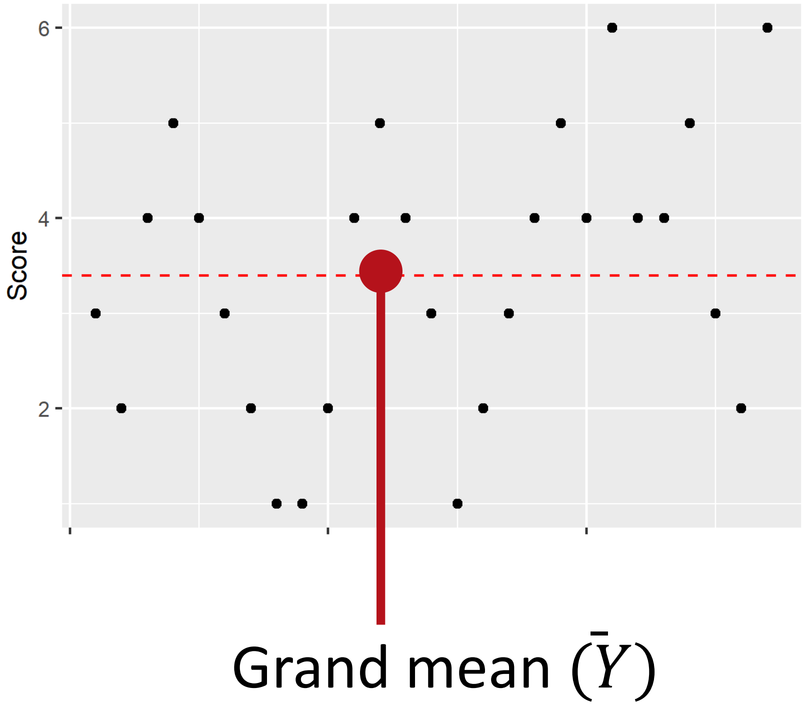
k = $number\ of\ levels$

$$\bar{Y} = \frac{2.78 + 3.11 + 4.33}{3}$$

$\bar{Y}$ = 3.41

Grand Mean ($\bar{Y}$)

A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
1	4	6	
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$	$\bar{Y} = 3.41$



Total between-group variance

$$total\ between\ group\ variance = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2\ (and\ so\ on)}{total\ between\ group\ degrees\ of\ freedom}$$



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
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$\bar{A}_1$= 2.78	$\bar{A}_2$= 3.11	$\bar{A}_3$= 4.33	$\bar{Y}$ = 3.41

Total between-group variance

$$total\ between\ group\ variance = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2\ (and\ so\ on)}{total\ between\ group\ degrees\ of\ freedom}$$

A_1 scores	A_2 scores	A_3 scores	
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1	3	2	
1	4	6	
$\bar{A}_1$= 2.78	$\bar{A}_2$= 3.11	$\bar{A}_3$= 4.33	$\bar{Y}$ = 3.41

Total between-group variance

$$\text{total between group variance} = \frac{N_{A1}(\bar{A}_1 - \bar{Y})^2 + N_{A2}(\bar{A}_2 - \bar{Y})^2 + N_{A3}(\bar{A}_3 - \bar{Y})^2\ (\text{and so on})}{\text{total between group degrees of freedom}}$$

N_{A1} = Number of scores for A_1 = 9

N_{A2} = Number of scores for A_2 = 9

N_{A3} = Number of scores for A_3 = 9



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
1	4	6	
$\bar{A}_1$ = 2.78	$\bar{A}_2$ = 3.11	$\bar{A}_3$ = 4.33	$\bar{Y}$ = 3.41

Degrees of freedom

Between-groups degrees of freedom

- The total number of levels minus one
- For example, in our experiment we have three levels [verbal feedback, written feedback, control]
- The between-groups degree of freedom is there 3 levels – 1 = 2
- Between-groups df = 2

Total between-group variance

$$total\ between\ group\ variance = \frac{9(2.78 - 3.41)^2 + 9(3.11 - 3.41)^2 + 9(4.33 - 3.41)^2}{2}$$

N_{A1} = Number of scores for A_1
= 9

N_{A2} = Number of scores for A_2
= 9

N_{A3} = Number of scores for A_3
= 9



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
1	4	6	
$\bar{A}_1$ = 2.78	$\bar{A}_2$ = 3.11	$\bar{A}_3$ = 4.33	$\bar{Y}$ = 3.41

Total between-group variance

$$total\ between\ group\ variance = \frac{3.60 + 0.81 + 7.65}{2} = 6.037\ (\text{with rounding})$$



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
1	4	6	
$\bar{A}_1 = 2.78$	$\bar{A}_2 = 3.11$	$\bar{A}_3 = 4.33$	$\bar{Y} = 3.41$

Calculating between-group variance

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{6.037}{\text{within-group variance}}$$

Total within-group variance

$$total\ within\ group\ variance = \frac{SS\ level\ A_1 + SS\ level\ A_2 + SS\ level\ A_3 (and\ so\ on)}{total\ within\ group\ degrees\ of\ freedom}$$

Total within-group variance

$$total\ within\ group\ variance = \frac{SS\ level\ A_1 + SS\ level\ A_2 + SS\ level\ A_3 (and\ so\ on)}{total\ within\ group\ degrees\ of\ freedom}$$

$SS\ level\ A_1 = Sums\ of\ squares\ for\ level\ 1$

$SS\ level\ A_2 = Sums\ of\ squares\ for\ level\ 2$

$SS\ level\ A_3 = Sums\ of\ squares\ for\ level\ 3$



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
1	4	6	
$\bar{A}_1$= 2.78	$\bar{A}_2$= 3.11	$\bar{A}_3$= 4.33	$\bar{Y}$ = 3.41

Total within-group variance

$$total\ within\text{-}group\ variance = \frac{\sum(A_1 - \bar{A}_1)^2 + (A_2 - \bar{A}_2)^2 + (A_3 - \bar{A}_3)^2 + \ (and\ so\ on)}{total\ within\ group\ degrees\ of\ freedom}$$

A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
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1	3	2	
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$\bar{A}_1$= 2.78	$\bar{A}_2$= 3.11	$\bar{A}_3$= 4.33	$\bar{Y}$ = 3.41

Total within-group variance

$$total\ within\ group\ variance = \frac{\sum(A_1 - 2.78)^2 + (A_2 - 3.11)^2 + (A_3 - 4.33)^2 + \ (and\ so\ on)}{total\ within\ group\ degrees\ of\ freedom}$$



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
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5	4	4	
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$\bar{A}_1$ = 2.78	$\bar{A}_2$ = 3.11	$\bar{A}_3$ = 4.33	$\bar{Y}$ = 3.41

Degrees of freedom

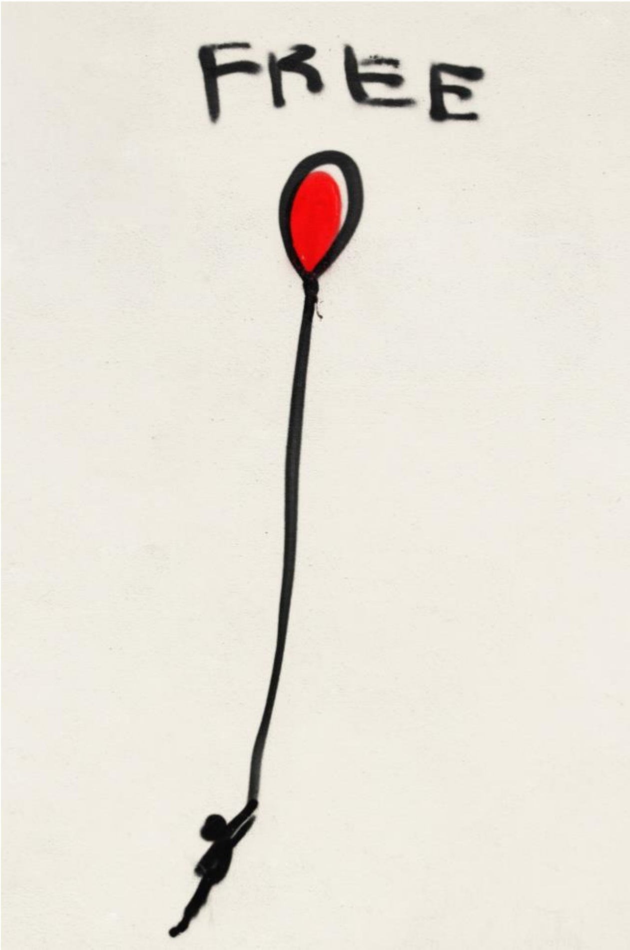
Within-groups degrees of freedom

- For within-groups degrees of freedom, we add up the number of participants for each level – 1
- Mathematically this is expressed as:

$$= (N_{A1} - 1) + (N_{A2} - 1) + (N_{A3} - 1)$$

$$= (9 \ - 1) + (9 - 1) + (9 \ - 1)$$

$$= 24$$



Total within-group variance

$$total\ within\ group\ variance = \frac{\sum(A_1 - 2.75)^2 + (A_2 - 3.11)^2 + (A_3 - 4.33)^2}{24}$$



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
1	4	6	
$\bar{A}_1$= 2.78	$\bar{A}_2$= 3.11	$\bar{A}_3$= 4.33	$\bar{Y}$ = 3.41

Total within-group variance

$$total\ within\ group\ variance = \frac{42.444}{24} = 1.769\ (with\ rounding)$$



A_1 scores	A_2 scores	A_3 scores	
3	2	5	
2	4	4	
4	5	6	
5	4	4	
4	3	4	
3	1	5	
2	2	3	
1	3	2	
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$\bar{A}_1$= 2.78	$\bar{A}_2$= 3.11	$\bar{A}_3$= 4.33	$\bar{Y}$ = 3.41

The F ratio

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{6.037}{1.769}$$

$F = 3.414$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251	252	253	254
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Source: E. S. Pearson and H. O. Hartley, *Biometrika Tables for Statisticians*, Vol. 2 (1972), Table 5, page 178, by permission.

Lancaster University

The F ratio

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{6.037}{1.769}$$

F = 3.414, *p* = 0.05, A statistically significant test result (P ≤ 0.05)


Savage Chickens
by Doug Savage
HAVE YOU READ MY BLOG YET?
WHY? I DON'T EVEN FIND YOU INTERESTING IN PERSON
©2005 BY DOUG SAVAGE
www.savagechickens.com

Assumptions of ANOVA and follow-up procedures

Agenda/Content for Lecture 3

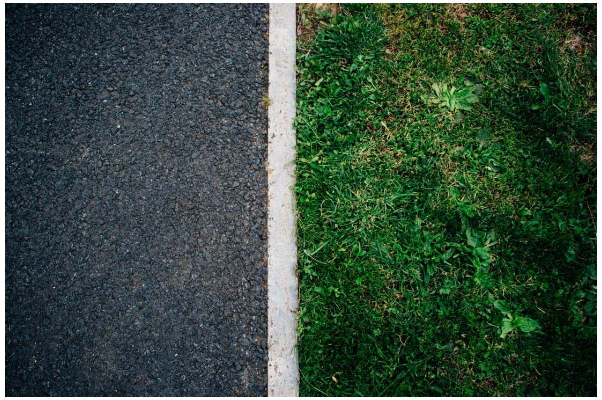
- Assumptions of ANOVA
  - Assumption of independence
  - Assumption of normality
  - Assumption of homogeneity of variance
- Data transformations
- Pairwise between-level comparisons
  - Planned comparisons
  - Post-hoc tests

In a perfect world…

- Normally distributed data
- You would have equal number of participants per level (e.g., per condition)
- Your data would be on an interval/ratio scale

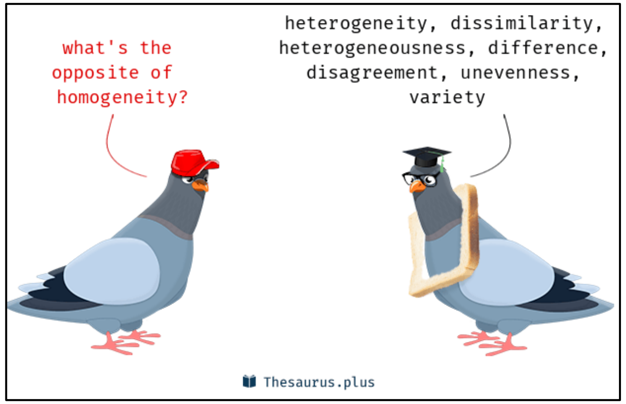
Assumptions underlying the ANOVA

1. Assumption of independence
2. Assumption of normality
3. Assumption of homogeneity of variance



Independence

Normality

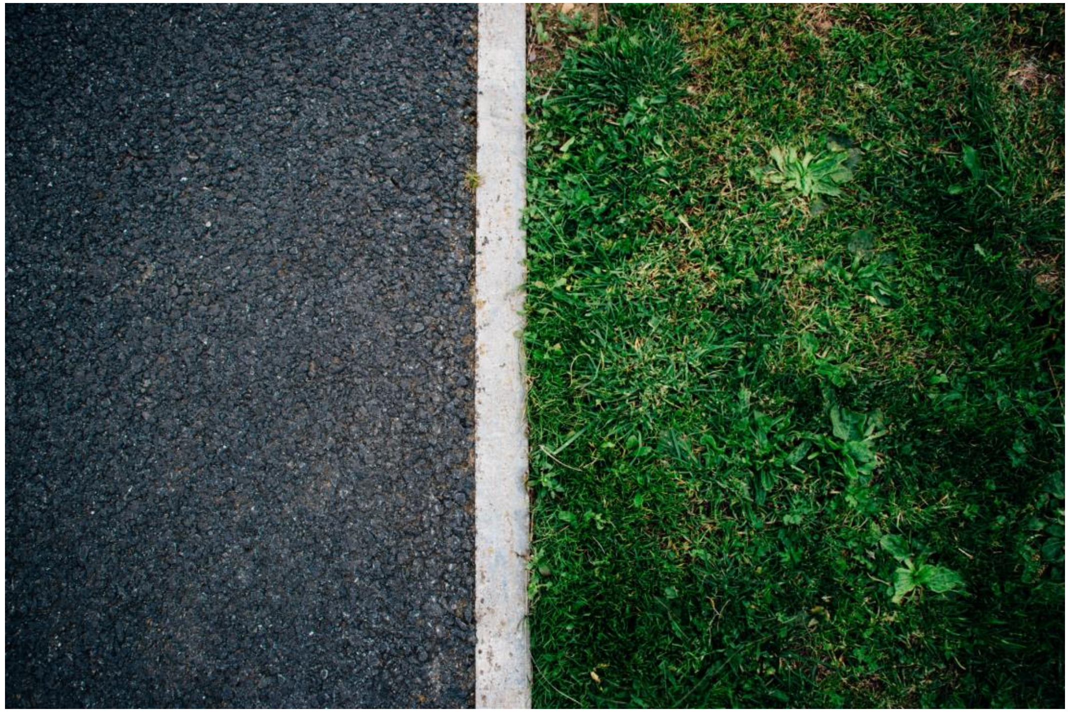


Homogeneity of variance

1. Assumption of independence

What is it?

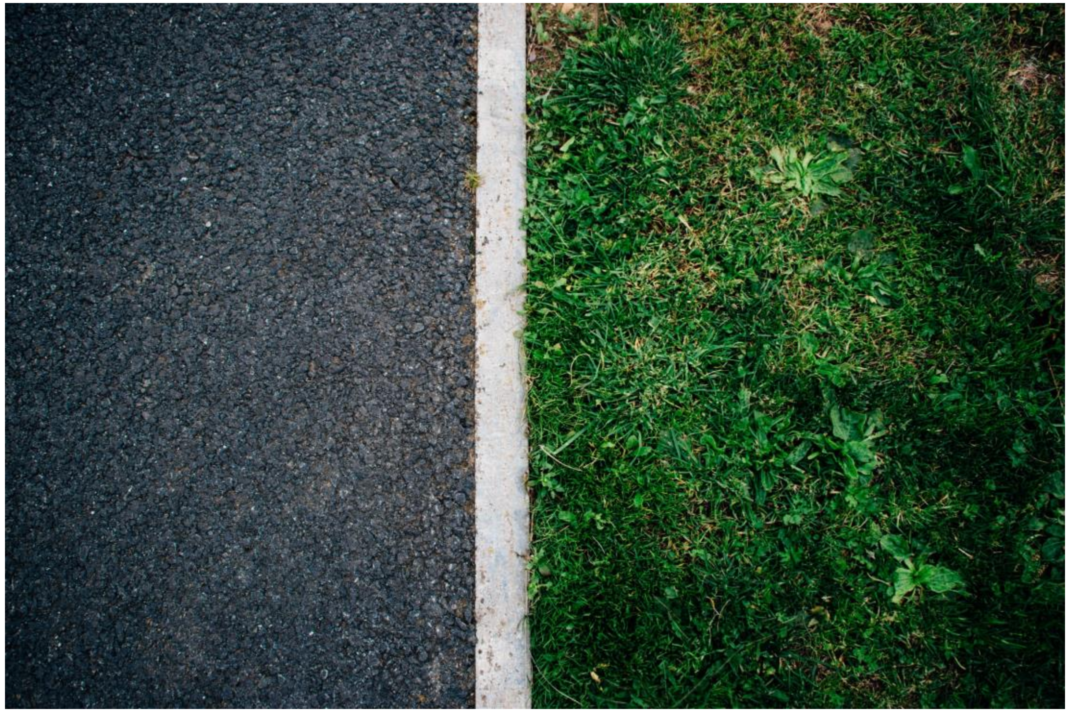
- Participants should be randomly assigned to a group
- Participants should not cluster, sharing a classification variable
  - Gender
  - Skill level
- There should be no influence across one data point to another



1. Assumption of independence

Consequences of violation

- Becomes difficult to interpret results
- Did the manipulation have an effect, or was this driven by classification clustering or influence?



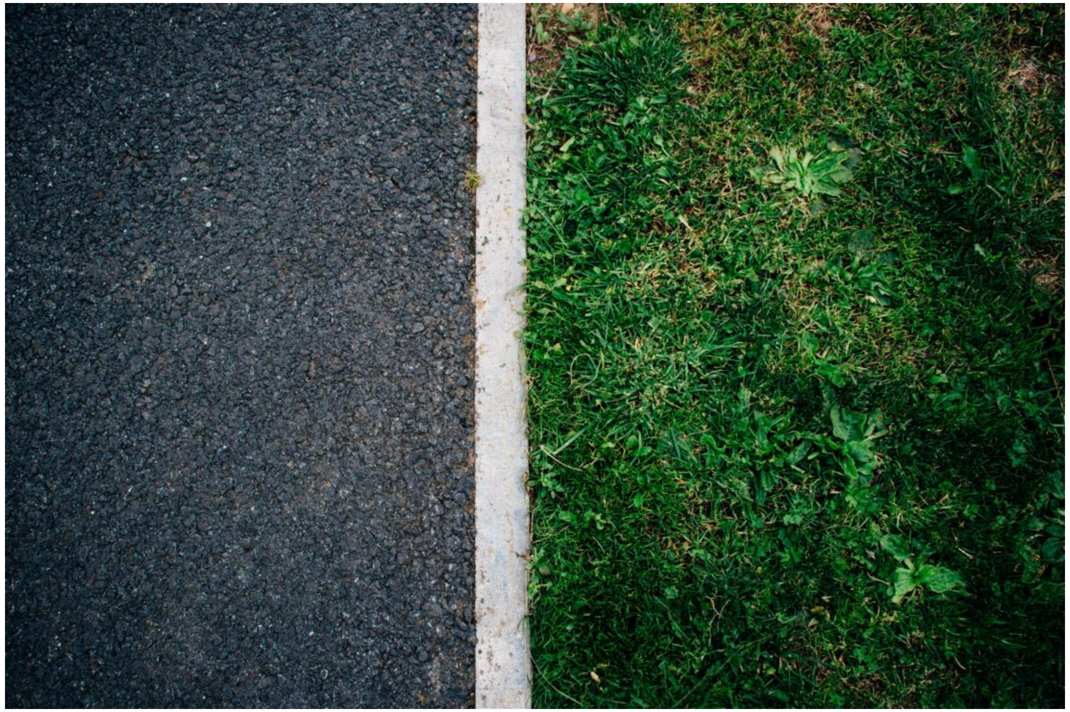
The F-ratio (from week 2!)

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

1. Assumption of independence

How to avoid it?

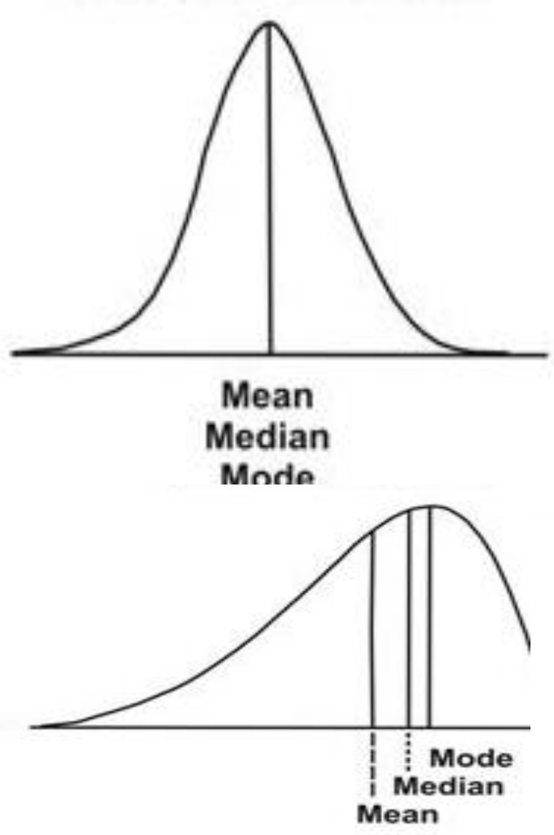
- Always randomly allocate participants to a condition
- Try to allocate equal numbers to each condition
- You can test to see whether you have significant differences on important classification variables



2. Assumption of normality

What is it?

- You want the overall data and the data for each subgroup to normally distributed
- This is because ANOVAs rely on the mean – and for skewed and bimodal data the mean is unlikely the best measure of central tendency



2. Assumption of normality

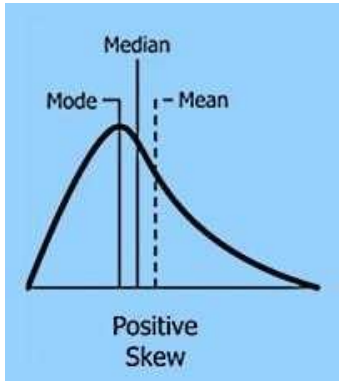
Consequences of violation

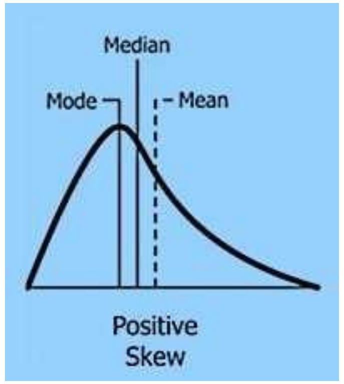
- If data are **slightly** skewed this is unlikely to cause problems

2. Assumption of normality

Consequences of violation

- If data are **slightly** skewed this is unlikely to cause problems
- If data are skewed by roughly the same degree in the same direction – unlikely a problem

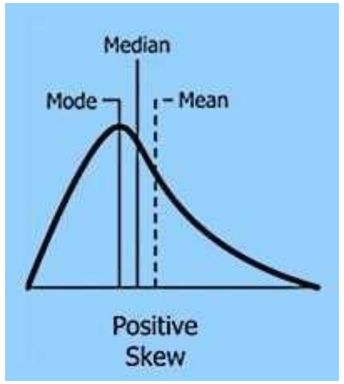


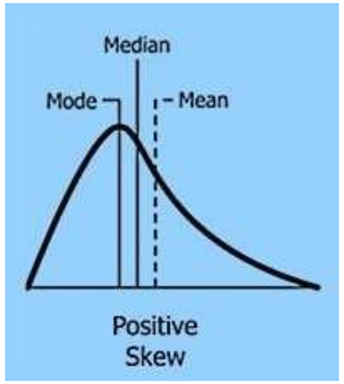


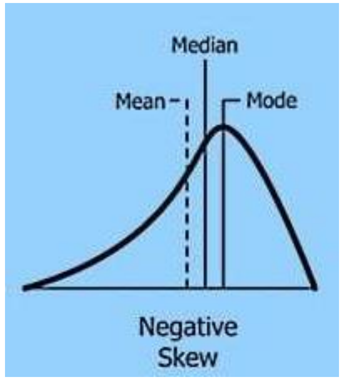
2. Assumption of normality

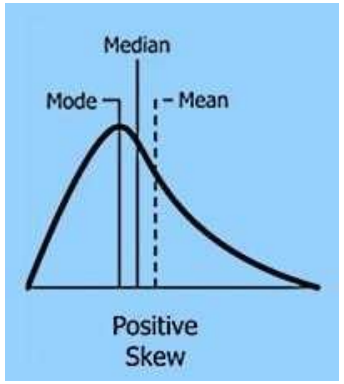
Consequences of violation

- If data are **slightly** skewed this is unlikely to cause problems
- If data are skewed by roughly the same degree in the same direction – unlikely a problem
- If skewed in different directions, this is a problem. Lead to type I and II errors!









2. Assumption of normality

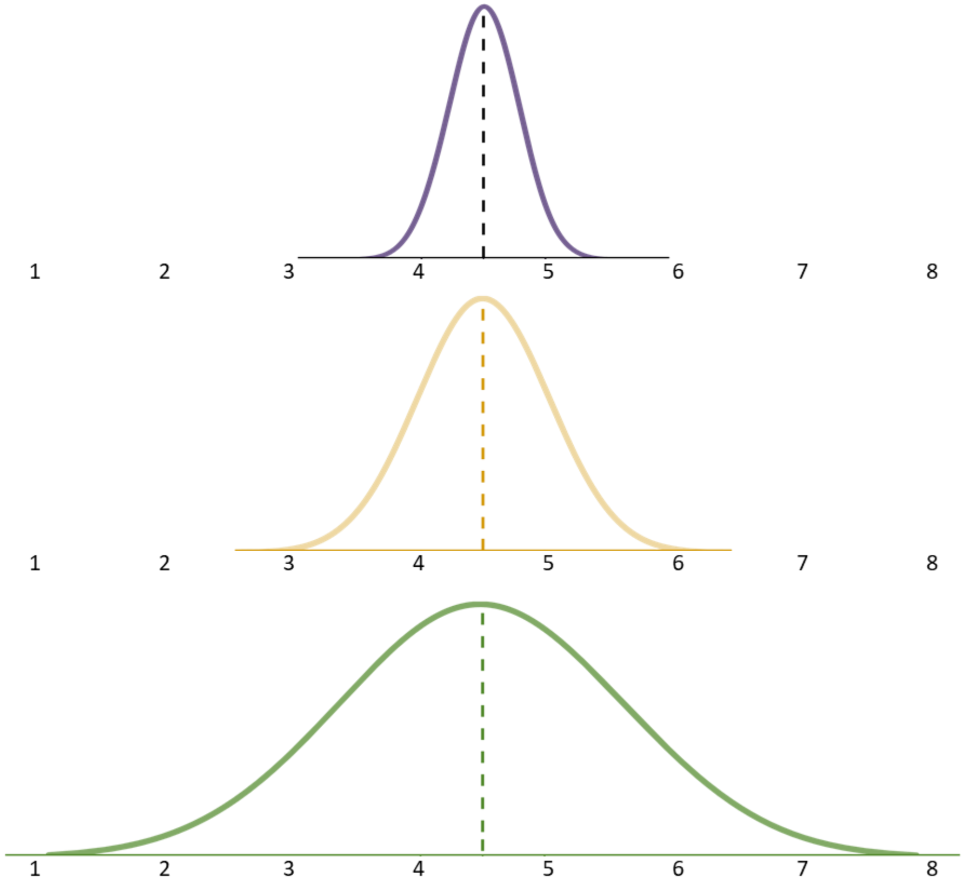
How to avoid it?

- Avoid measures which often have ceiling or floor effects
- Transform data, changing every score in a systematic way
- Use a robust ANOVA (specialized test – more complex) or non-parametric alternatives:
  The Kruskal-Wallis Test

3. Homogeneity of variance

What is it?

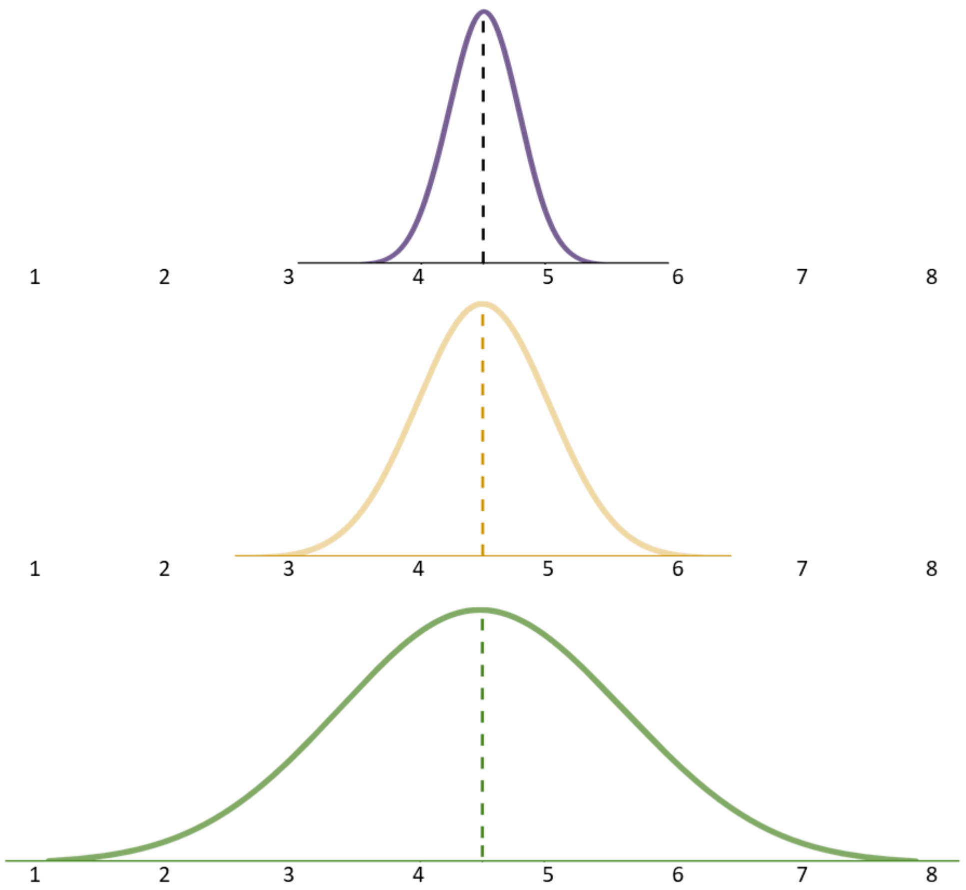
- Assumes that the variances of the distributions in the samples are equal
- Therefore the variances for each sample should not significantly vary from one another



3. Homogeneity of variance

Consequences of violation

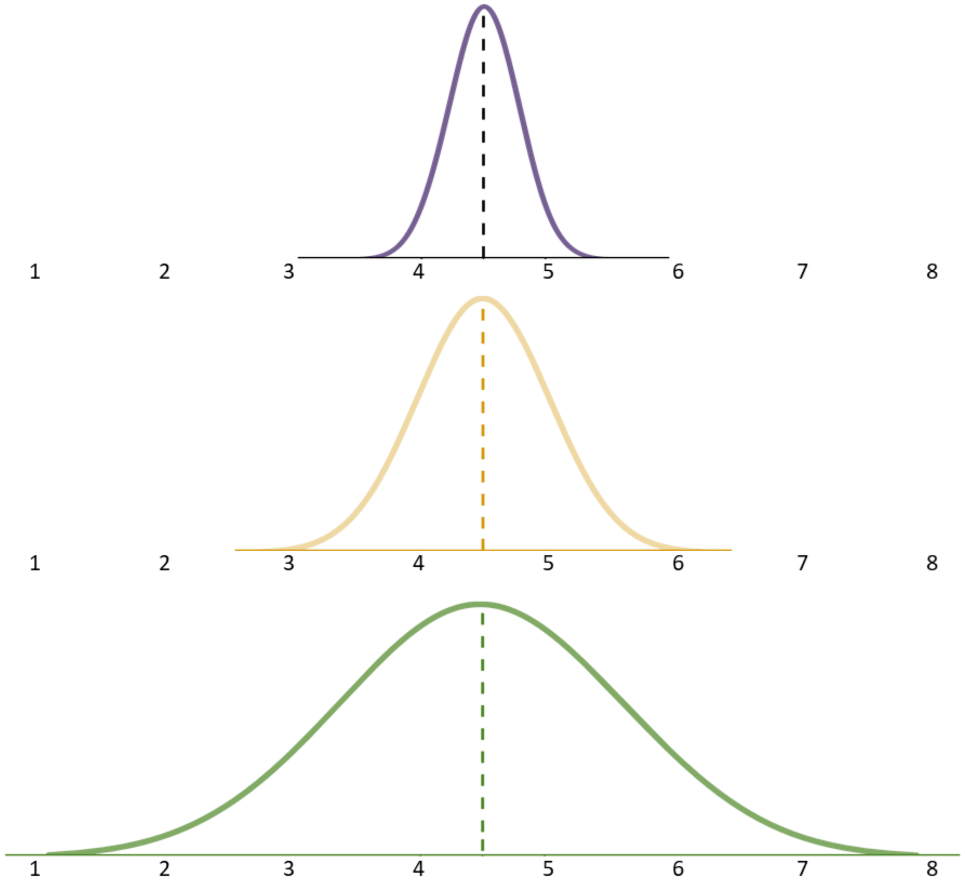
- The ANOVA tests the plausibility of the null hypothesis – i.e., all observations come from the same underlying population with the same degree of variability
- This is pointless to test when variance is already clearly different



3. Homogeneity of variance

How to avoid it?

- Difficult to avoid, but can be mitigated when testing
- As a rule of thumb, it is ok, as long as largest variance is no more than 4x the size of smallest
- Can also transform data or use non-parametric alternative



Dealing with ‘rogue’ data

Transforming data

- This involves taking every score from each participant and applying a uniform mathematical function to each
- Report both the original data and the transformed data

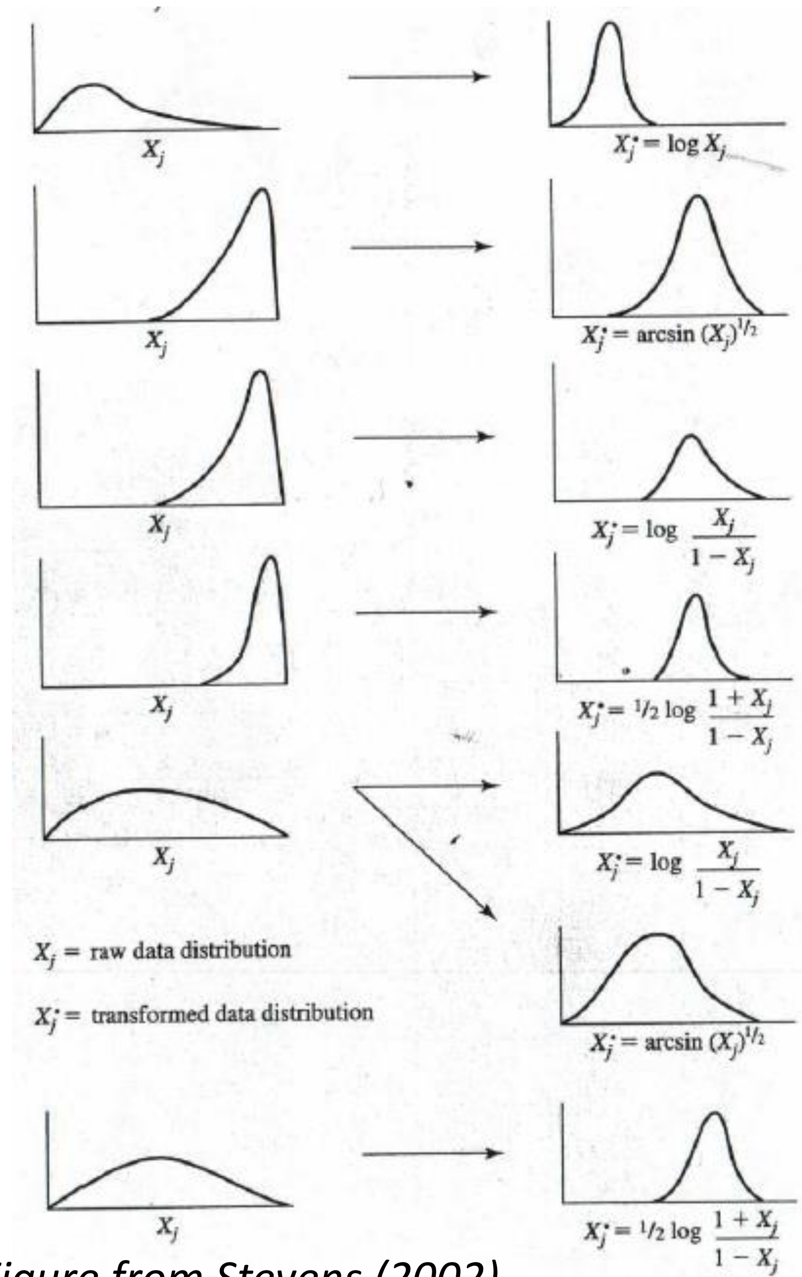


Figure from Stevens (2002)

Dealing with 'rogue' data

How to transform data

Untransformed	Square-root transformed	Log transformed
38	6.164	1.580
1	1.000	0.000
13	3.606	1.114
2	1.414	0.301
13	3.606	1.114
20	4.472	1.301
50	7.071	1.699
9	3.000	0.954
28	5.292	1.447
6	2.449	0.778
4	2.000	0.602
43	6.557	1.633

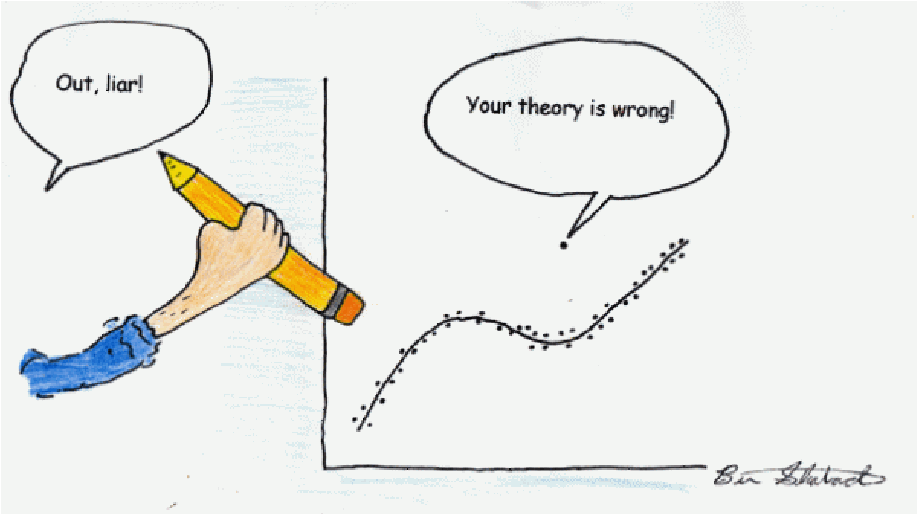
http://www.biostathandbook.com/transformation.html

Type of Data Transformation	Nature of Data
Log Transformation $(\log(X_i))$	Whole numbers and cover wide range of values, small values with decimal fractions.
Square-root Transformation $(\sqrt{X_i})$	Small whole number & Percentage data where the range is between 0 and 30 % or between 70 and 100 %

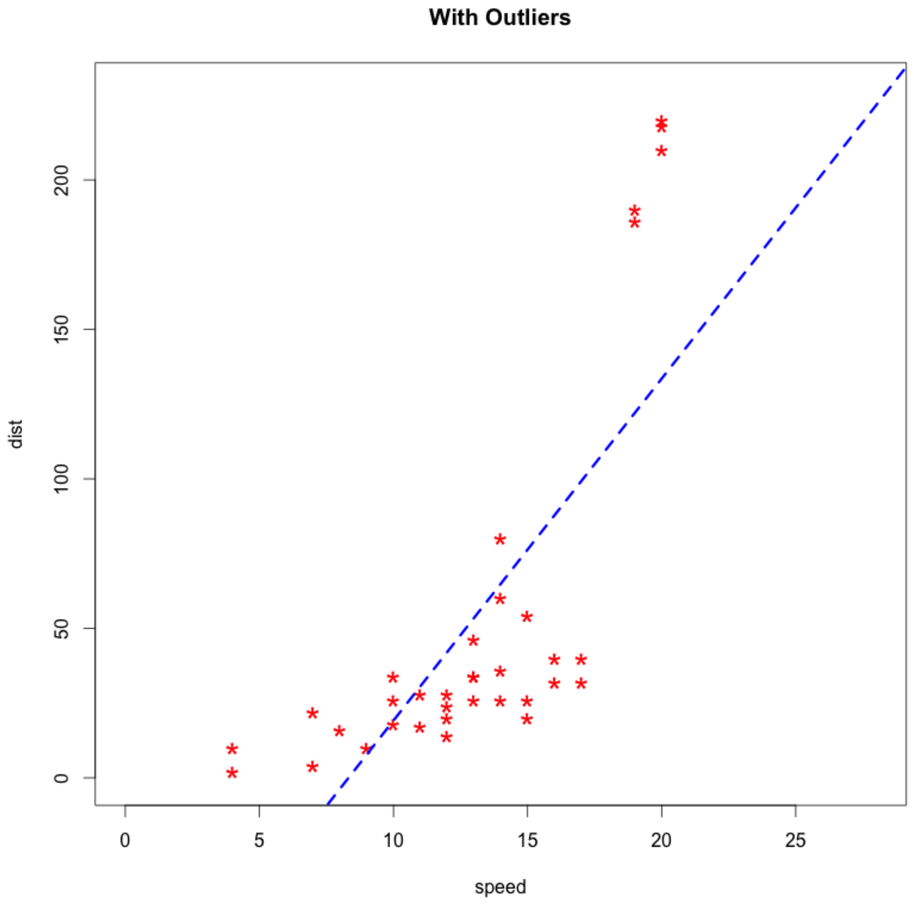
Maidapwad & Sananse (2014)

Outliers and their impact

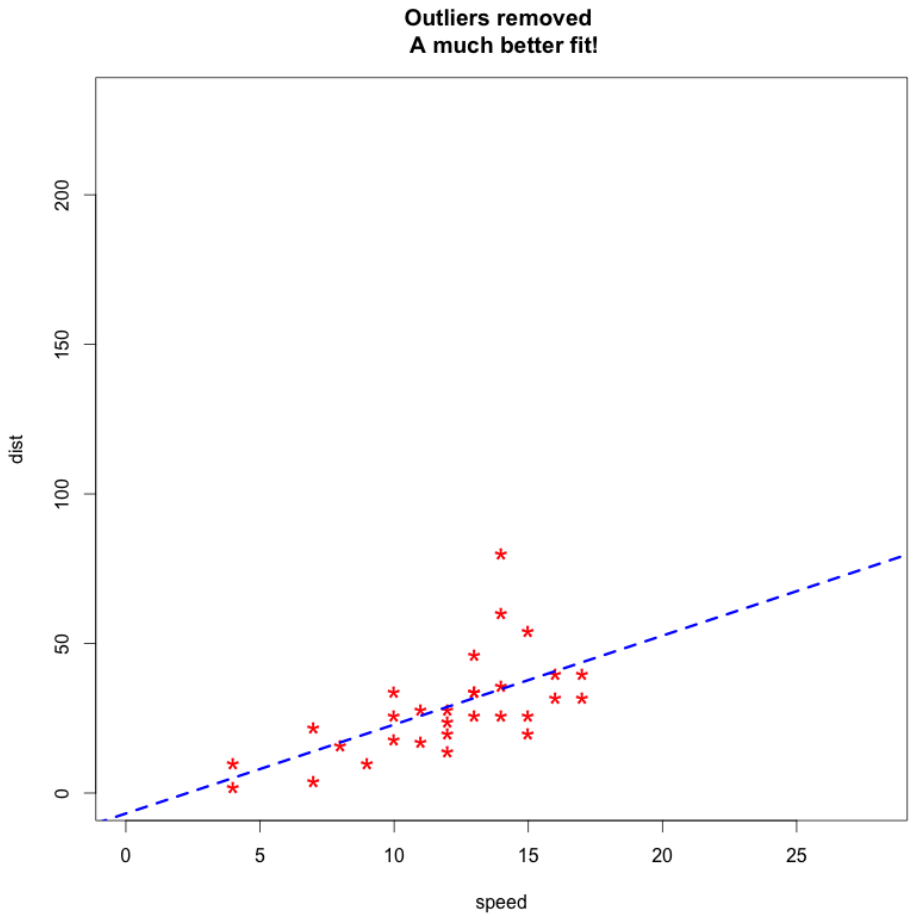
- Outliers are data points which significantly differ from other observations
- Outliers can drastically bias/change predictive models
- Predictions can be exaggerated and present high error
- Outliers not only distort statistical analyses, they can violate assumptions



Outliers and their impact



Outliers and their impact



- Given the problems outliers create, it may seem levelheaded to remove them
- However, it can be dishonest and misleading to do so if they are true scores
- It must be justifiable as to why it is necessary to remove data

The meaning of an ANOVA output

```
##              Df Sum Sq Mean Sq F value   Pr(>F)
## Group         2   1223   611.3   12.52 6.77e-06 ***
## Residuals   237  11571    48.8
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{611.3}{48.8}$$

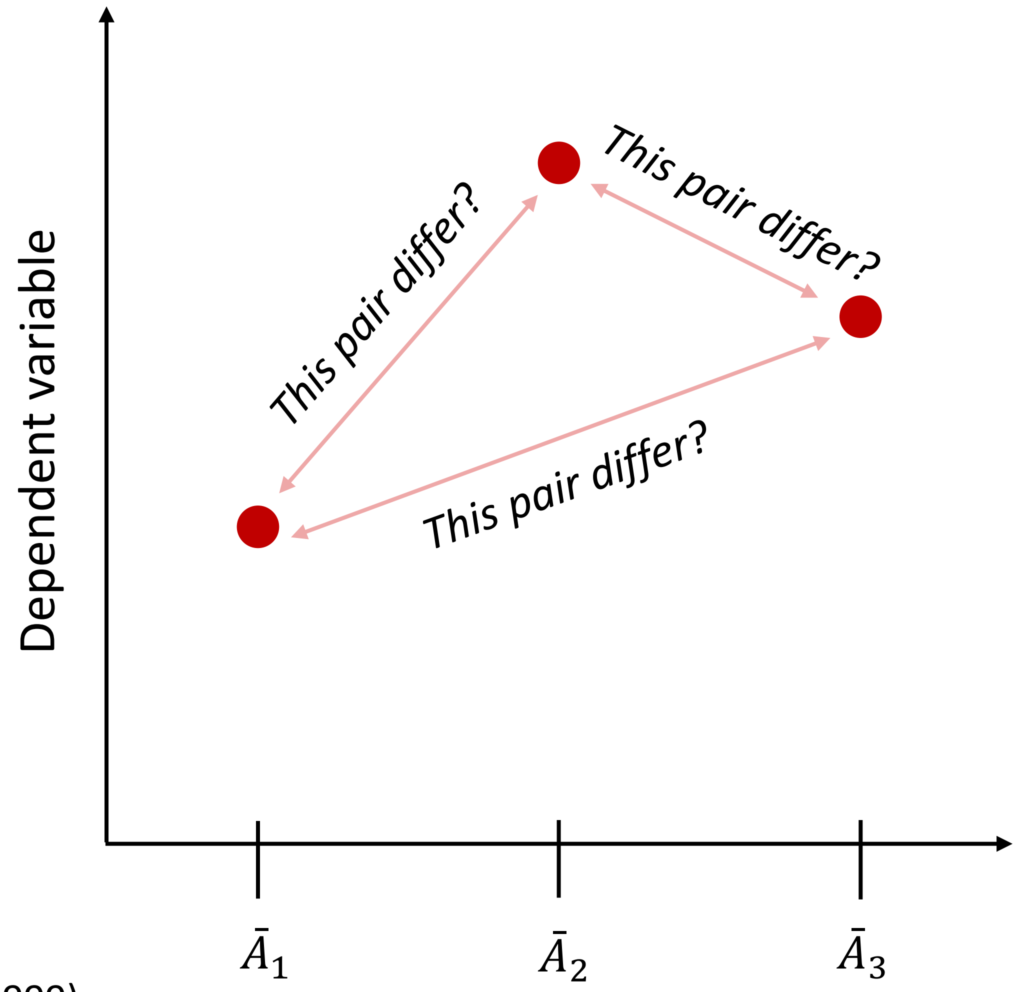
$F = 12.52$

$p = 0.00000677$

The meaning of an ANOVA output

P-value	Definition
> .05	▪ We accept the null hypothesis (H_0) ▪ Under H_0, the samples come from the same population ▪ There is no statistical difference in the population means ($\mu_1 = \mu_2 = \mu_3$) ▪ Experimental effect = 0
≤ .05	▪ We reject the null hypothesis (H_1) ▪ Under H_1, the samples come from different populations ▪ Population means are statistically different ($\mu_1 \neq \mu_2 \neq \mu_3$) ▪ Experimental effect ≠ 0

Significant



$p \leq .05$
At least one of the pairs of means is significantly different. The question is, which pairs?

Adapted from Roberts and Russo (1999)

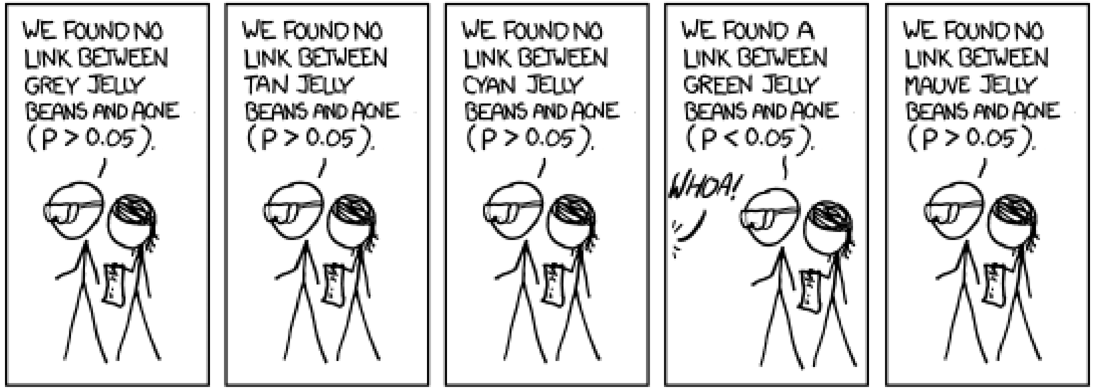
Pairwise comparisons

There are two strategies for following-up significant ANOVAs

- Planned comparisons
- Post-hoc comparisons

The problem of multiple comparisons

- Why not just run a bunch of t-tests?
- Multiple comparisons increase the probability of making a (familywise) type I error
- I.e., rejecting the null hypothesis when actually there was no effect



The problem of multiple comparisons

- Type 1 error - 1 test at $p \leq 0.05$ = 0.95 (i.e., 5% chance we get noise)
- Type 1 error - 2 tests = 0.95 * 0.95, = 0.903. (10% chance)
- Type 1 error - 3 tests = 0.95 * 0.95 * 0.95 = 0.857 (14% chance)
- Type 1 error – 4 tests = 0.95 * 0.95 * 0.95 * 0.95 = 0.815 (18.5% chance)
- Type 1 error – 5 tests = 0.95 * 0.95 * 0.95 * 0.95 * 0.95 = 0.774 (22.6% chance)

Planned comparisons

- Focussed approach to examine specific group differences
- Perfect when certain hypotheses can be tested without comparing all combinations of means
- Should be pre-specified
- Need to keep the number of planned comparisons as low as possible to negate Type I errors – (number of levels – 1)

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

Planned comparisons

Our options:

1. Run t-tests with a low number of pairs
2. Run t-tests with Bonferroni adjustment
3. ~~Specialized linear contrast~~

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

Planned comparisons – 2. Corrections

- Continue to run t-tests, but adjust the *p* value to make it more conservative
- Only accept significant if below this threshold
- Bonferroni Correction:
  - A new p-value is generated from the prior significance level divided by the number of tests

$$0.05 \div 2 = 0.025$$

P-value (0.05)

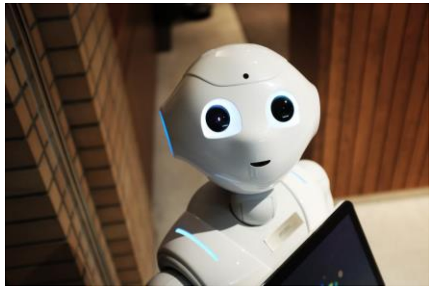
Number of tests (2)

Bonferroni adjusted P-value (0.025)

Planned comparisons – 1. Run t-tests

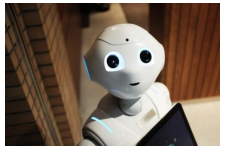
A_1 - Robot A(lpha)

$$t = \frac{\bar{A}_1 - \bar{A}_2}{\sqrt{(Mean\ Square_{ERROR})(\frac{2}{NA})}}$$



A_2- Robot B(eta)

Planned comparisons – 2. Corrections

$t =$ -2.94, with 237 degrees of freedom

It's significant at $p = 0.025$ threshold

Degrees of Freedom	p=0.05	p=0.025	p=0.01	p=0.005
1	12.71	25.45	63.66	127.32
2	4.30	6.20	9.92	14.09
3	3.18	4.17	5.84	7.45
4	2.78	3.50	4.60	5.60
5	2.57	3.16	4.03	4.77
6	2.45	2.97	3.71	4.32
7	2.36	2.84	3.50	4.03
8	2.31	2.75	3.36	3.83
9	2.26	2.68	3.25	3.69
10	2.23	2.63	3.17	3.58
11	2.20	2.59	3.11	3.50
12	2.18	2.56	3.05	3.43
13	2.16	2.53	3.01	3.37
14	2.14	2.51	2.98	3.33
15	2.13	2.49	2.95	3.29
16	2.12	2.47	2.92	3.25
17	2.11	2.46	2.90	3.22
18	2.10	2.44	2.88	3.20
19	2.09	2.43	2.86	3.17
20	2.09	2.42	2.84	3.15
21	2.08	2.41	2.83	3.14
22	2.07	2.41	2.82	3.12
23	2.07	2.40	2.81	3.10
24	2.06	2.39	2.80	3.09
25	2.06	2.38	2.79	3.08
26	2.06	2.38	2.78	3.07
27	2.05	2.37	2.77	3.06
28	2.05	2.37	2.76	3.05
29	2.04	2.36	2.76	3.04
30	2.04	2.36	2.75	3.03
40	2.02	2.33	2.70	2.97
60	2.00	2.30	2.66	2.92
120	1.98	2.27	2.62	2.86
infinity	1.96	2.24	2.58	2.81

Post hoc tests

- Post hoc comes from Latin for “after the event”
- Post hoc tests assess all possible combinations of differences between groups by comparing each mean with the other
- Make adjustments to *p* value, but more conservative than Bonferroni correction

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-

Post hoc tests

Method	Equal N F	Normality	Use	Error control	Protection
Fisher PLSD	Yes	Yes	Yes	All	Most sensitive to Type 1
Tukey-Kramer HSD	No	Yes	Yes	All	Less sensitive to Type 1 than Fisher PLSD
Spjotvoll-Stoline	No	Yes	Yes	All	As Tukey-Kramer
Student-Newman Keuls (SNK)	Yes	Yes	Yes	All	Sensitive to Type 2
Tukey-Compromise	No	Yes	Yes	All	Average of Tukey and SNK
Duncan's Multiple Range	No	Yes	Yes	All	More sensitive to Type 1 than SNK
Scheffé's S	Yes	No	No	All	Most conservative
Games/Howell	Yes	No	No	All	More conservative than majority
Dunnett's test	No	No	No	T/C	More conservative than majority
Bonferroni	No	Yes	Yes	All, TC	Conservative

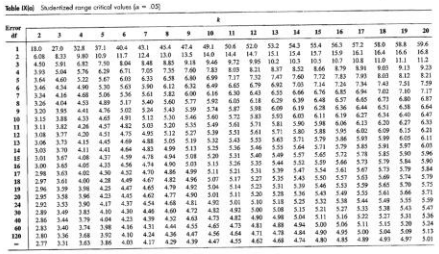
https://www.researchgate.net/profile/Cyril-Iaconelli/post/The_choice_of_post-hoc_test/

Post hoc tests – Tukey-Kramer HSD



Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$
 $\bar{A}_1$	-	-	-
 $\bar{A}_2$	●	-	-
 $\bar{A}_3$	●	●	-

Table IX: Tukey α = 0.05



Studentized range statistic [num means, df]

$$W = q_{(r, df_{ERROR})} \sqrt{\frac{Mean\ Square_{ERROR}}{N_A}}$$

Within group variance from ANOVA output

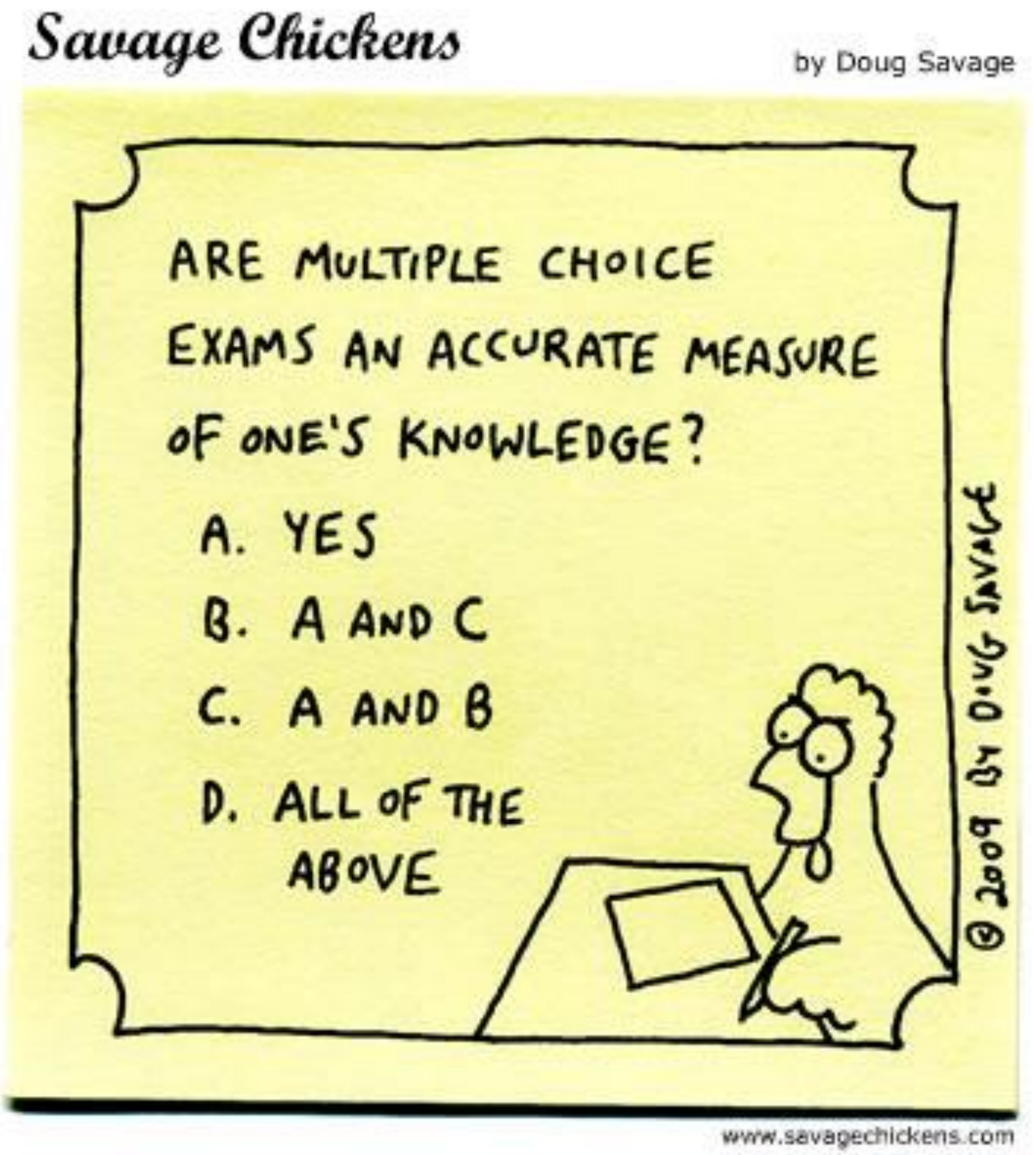
Number of participants

Post hoc tests – Tukey-Kramer HSD

Take home message:

- As you add more and more mean comparisons, you require larger critical values (q) in the standardized table to find a statistical difference!
- As such, test what you need, not what you don't!

Group	$\bar{A}_1$	$\bar{A}_2$	$\bar{A}_3$	$\bar{A}_4$	$\bar{A}_5$
$\bar{A}_1$	-	-	-	-	-
$\bar{A}_2$	●	-	-	-	-
$\bar{A}_3$	●	●	-	-	-
$\bar{A}_4$	●	●	●	-	-
$\bar{A}_5$	●	●	●	●	-



Savage Chickens
by Doug Savage
ARE MULTIPLE CHOICE EXAMS AN ACCURATE MEASURE OF ONE'S KNOWLEDGE?
A. YES
B. A AND C
C. A AND B
D. ALL OF THE ABOVE
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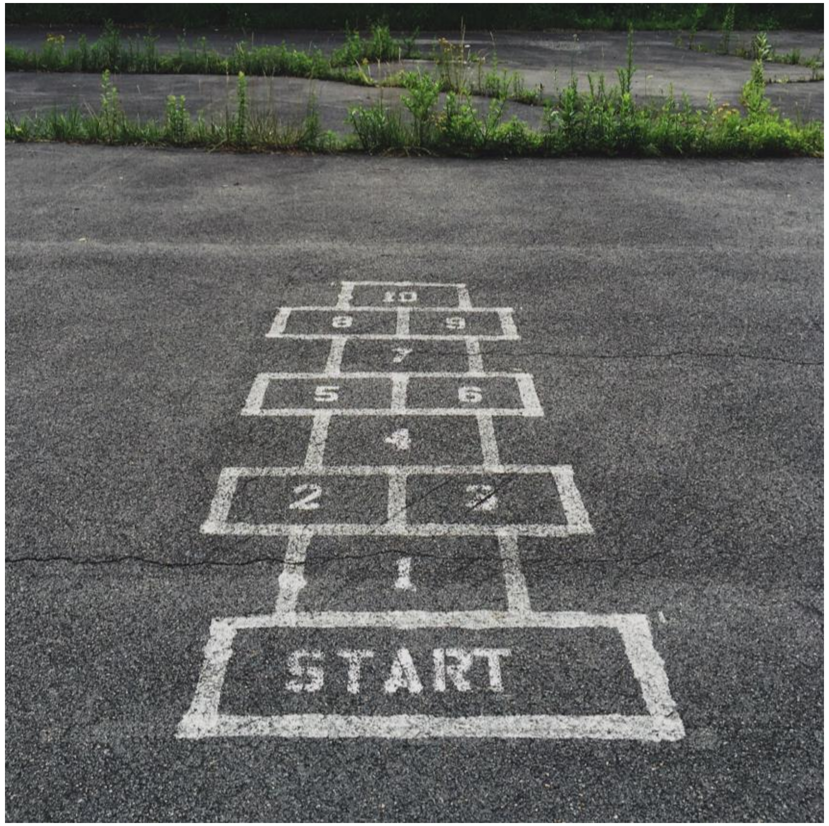
One factor within-participants ANOVA

Agenda/Content for Lecture 4

- Introduction to one factor within-participants ANOVA and its limitations
- Between-participant variability and residual variance
- Calculating within-group and between group variances
- Producing the within-participants F-statistic



Within-participants

Within-participants design - limitations

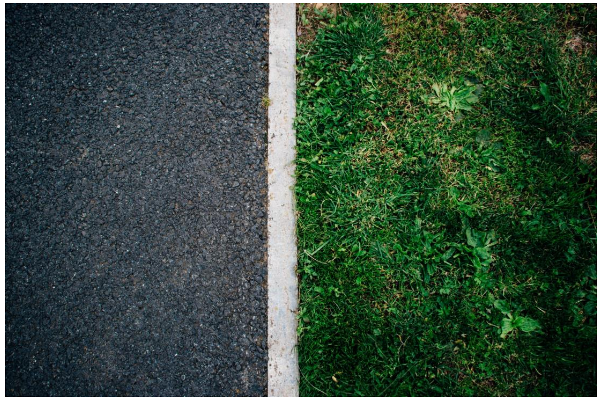
Lancaster University

	Type	Definition	An example...
Order effects	Practice effects	The experience/performance on a task at a given point in time, may influence your performance of that task at a subsequent time.	
	Fatigue effects	Fatigue or boredom with a task may influence your performance of that task at a subsequent time.	
	Demand characteristic	Participants form an idea of the experiment's purpose and (sub)consciously change their behaviour to comply	

Assumptions underlying the W-P ANOVA

1. Assumption of independence
2. Assumption of normality
3. Assumption of **sphericity**

The variances of the differences between all combinations of related groups are equal



Independence

Normality

Sphericity

Within-participants F ratio

$$F = \frac{\text{between-group variance}}{\text{within-group variance}}$$

$$F = \frac{\text{treatment effects + random (residual) errors}}{\text{random (residual) errors}}$$

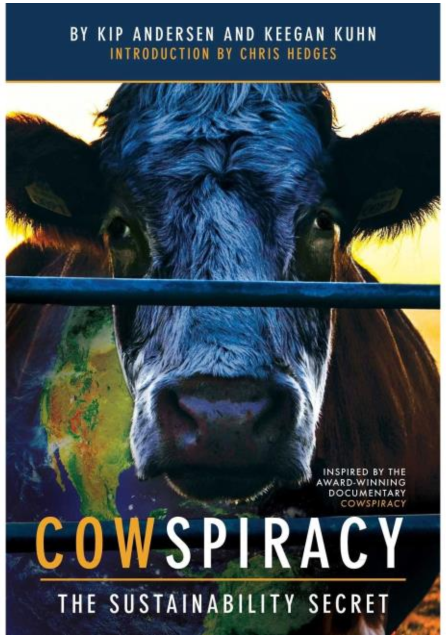
The F ratio

$F = \dfrac{\text{Signal}}{\textbf{Noise}}$

$F = \dfrac{\textbf{Signal}}{\text{Noise}}$

The larger in magnitude the F value, the more treatment effects are standing out away from experimental error – i.e., the larger the signal is from the noise. The larger the F, the less likely that differences in scores are caused by chance.

A within-participants example



Summary

Table 5. Burgers consumed before (A_1) and after (A_2) Cowspiracy

	A_1	A_2	ΔA	*P Mean*
P_1	5	3	-2	*4*
P_2	9	7	-2	*8*
P_3	3	1	-2	*2*
P_4	7	5	-2	*6*
P_5	4	6	2	*5*
A Mean	5.6	4.4		*5*

High between-participant variability / **Low** residual variance

Table 6. Burgers consumed before (A_1) and after (A_2) Cowspiracy

	A_1	A_2	ΔA	*P Mean*
P_1	9	1	-8	*5*
P_2	5	5	0	*5*
P_3	4	6	2	*5*
P_4	6	4	-2	*5*
P_5	4	6	2	*5*
A Mean	5.6	4.4		*5*

Low between-participant variability / **High** residual variance

Ingredients of within-participants ANOVA

Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{BETWEEN} = \frac{(\Sigma \mathrm{A}_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma \mathrm{A}_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

SS-Between groups

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

SS-Between groups



Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

$$SS_{BETWEEN} = \frac{(20)^2 + (41)^2 + (48)^2}{9} - \frac{(109)^2}{27}$$

$$SS_{BETWEEN} = \frac{400 + 1681 + 2304}{9} - \frac{11881}{27}$$

$$SS_{BETWEEN} = 44.44 + 186.77 + 256.00 - 440.03$$

$$SS_{BETWEEN} = 487.21 - 440.03$$

$$SS_{BETWEEN} = 47.18$$

Ingredients of within-participants ANOVA



Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{BETWEEN} = 47.18$$

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

Ingredients of within-participants ANOVA

Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$

SS-Within group

Participant	A_1^2 scores	A_2^2 scores	A_3^2 scores
1	$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
2	$1^2 = 1$	$4^2 = 16$	$4^2 = 16$
3	$3^2 = 9$	$5^2 = 25$	$6^2 = 36$
4	$2^2 = 4$	$6^2 = 36$	$5^2 = 25$
5	$2^2 = 4$	$3^2 = 9$	$3^2 = 9$
6	$1^2 = 1$	$5^2 = 25$	$6^2 = 36$
7	$4^2 = 16$	$7^2 = 49$	$7^2 = 49$
8	$3^2 = 9$	$3^2 = 9$	$6^2 = 36$
9	$2^2 = 4$	$5^2 = 25$	$6^2 = 36$
Total	20	41	48

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$

$$SS_{WITHIN} = 523 - \frac{(20)^2 + (41)^2 + (48)^2}{9}$$

$$SS_{WITHIN} = 523 - \frac{400 + 1681 + 2304}{9}$$

$$SS_{WITHIN} = 523 - 487.21$$

$$SS_{WITHIN} = 35.79$$

Ingredients of within-participants ANOVA

Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
$Total$	20	41	48

$$SS_{BETWEEN} = 47.18$$

$$SS_{WITHIN} = 35.79$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

Ingredients of within-participants ANOVA



Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
Total	20	41	48

$$SS_{BETWEEN} = \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A} - \frac{(\Sigma Y)^2}{N}$$

$$SS_{WITHIN} = \Sigma Y^2 - \frac{(\Sigma A_1)^2 + (\Sigma A_2)^2 + (\Sigma A_3)^2}{N_A}$$

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

SS-Total

Participant	$A_1{}^2$ scores	$A_2{}^2$ scores	$A_3{}^2$ scores
1	$2^2 = 4$	$3^2 = 9$	$5^2 = 25$
2	$1^2 = 1$	$4^2 = 16$	$4^2 = 16$
3	$3^2 = 9$	$5^2 = 25$	$6^2 = 36$
4	$2^2 = 4$	$6^2 = 36$	$5^2 = 25$
5	$2^2 = 4$	$3^2 = 9$	$3^2 = 9$
6	$1^2 = 1$	$5^2 = 25$	$6^2 = 36$
7	$4^2 = 16$	$7^2 = 49$	$7^2 = 49$
8	$3^2 = 9$	$3^2 = 9$	$6^2 = 36$
9	$2^2 = 4$	$5^2 = 25$	$6^2 = 36$
Total	20	41	48

$$SS_{TOTAL} = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N}$$

$$SS_{TOTAL} = 523 - \frac{(109)^2}{27}$$

$$SS_{TOTAL} = 523 - \frac{11881}{27}$$

$$SS_{TOTAL} = 523 - 440.03$$

$$SS_{TOTAL} = 82.97$$

Ingredients of within-participants ANOVA

Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
$Total$	20	41	48

$SS_{BETWEEN} = 47.18$

$SS_{WITHIN} = 35.79$

$SS_{TOTAL} = 82.97$

Ingredients of within-participants ANOVA



Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
$Total$	20	41	48

$SS_{BETWEEN} = 47.18$

$SS_{WITHIN} = 35.79$

$SS_{TOTAL} = 82.97$

$$SS_{between\ participants} = \frac{(\Sigma P_1)^2 + (\Sigma P_2)^2\ (and\ so\ on)}{N_P} - \frac{(\Sigma Y)^2}{N}$$

SS-between participants

$$SS_{between\ participants} = \frac{(\Sigma P_1)^2 + (\Sigma P_2)^2\ (and\ so\ on)}{N_P} - \frac{(\Sigma Y)^2}{N}$$



Participant	A_1 scores	A_2 scores	A_3 scores	P total	
1	2	3	5	10	
2	1	4	4	9	
3	3	5	6	14	
4	2	6	5	13	
5	2	3	3	8	
6	1	5	6	12	
7	4	7	7	18	
8	3	3	6	12	
9	2	5	6	13	
Total	20	41	48	109	

SS-between participants



$$SS_{between\ participants} = \frac{(\Sigma P_1)^2 + (\Sigma P_2)^2\ (and\ so\ on)}{N_P} - \frac{(\Sigma Y)^2}{N}$$

Participant	A_1 scores	A_2 scores	A_3 scores	P total
1	2	3	5	10
2	1	4	4	9
3	3	5	6	14
4	2	6	5	13
5	2	3	3	8
6	1	5	6	12
7	4	7	7	18
8	3	3	6	12
9	2	5	6	13
Total	20	41	48	109

$$\left(\frac{10^2}{3} + \frac{9^2}{3} + \frac{14^2}{3} + \frac{13^2}{3} + \frac{8^2}{3} + \frac{12^2}{3} + \frac{18^2}{3} + \frac{12^2}{3} + \frac{13^2}{3}\right) - \frac{(109)^2}{27}$$

$$\left(\frac{100}{3} + \frac{81}{3} + \frac{196}{3} + \frac{169}{3} + \frac{64}{3} + \frac{144}{3} + \frac{324}{3} + \frac{144}{3} + \frac{169}{3}\right) - \frac{(109)^2}{27}$$

$$(33.33 + 27 + 65.33 + 56.33 + 21.33 + 48 + 108 + 48 + 56.33)$$

- 440.03

463.67 − 440.03

= 23.64

Ingredients of within-participants ANOVA

Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
$Total$	20	41	48

$SS_{BETWEEN} = 47.18$

$SS_{WITHIN} = 35.79$

$SS_{TOTAL} = 82.97$

$SS_{between\ participants} = 23.64$

$SS_{RESIDUAL}$ ....

What we'll need for the ANOVA

$$SS_{RESIDUAL} = SS_{WITHIN} - SS_{between\ participants}$$

$$12.15 = 35.79 - 23.64$$

Ingredients of within-participants ANOVA

Lancaster University



Participant	A_1 scores	A_2 scores	A_3 scores
1	2	3	5
2	1	4	4
3	3	5	6
4	2	6	5
5	2	3	3
6	1	5	6
7	4	7	7
8	3	3	6
9	2	5	6
$Total$	20	41	48

$SS_{BETWEEN} = 47.18$

$SS_{WITHIN} = 35.79$

$SS_{TOTAL} = 82.97$

$SS_{between\ participants} = 23.64$

$SS_{RESIDUAL}$ …. = 12.15

What we'll need for the ANOVA

$$F = \frac{\text{between-group variance}}{\text{residual variance}}$$

$$\text{between-group variance} = \frac{SS_{BETWEEN}}{df_{BETWEEN}} = \frac{47.18}{2} = 23.59$$

a – 1 [i.e., number of levels – 1]

What we'll need for the ANOVA

$$F = \frac{23.59}{\text{residual variance}}$$

$$\text{between-group variance} = \frac{SS_{BETWEEN}}{df_{BETWEEN}} = \frac{47.18}{2} = 23.59$$

$$\text{residual variance} = \frac{SS_{RESIDUAL}}{df_{RESIDUAL}} = \frac{12.15}{16} = 0.76$$

(a – 1)*(p – 1)
[i.e., (no. of levels – 1) x (np. Participants – 1)]

What we'll need for the ANOVA

$$F = \frac{23.59}{0.76} = 31.04$$

	DF1	α = 0.05																
DF2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	Inf
1	161.45	199.5	215.71	224.58	230.16	233.99	236.77	238.88	240.54	241.88	243.91	245.95	248.01	249.05	250.1	251.14	252.2	253.25	254.31
2	18.513	19	19.164	19.247	19.296	19.33	19.353	19.371	19.385	19.396	19.413	19.429	19.446	19.454	19.462	19.471	19.479	19.487	19.496
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855	8.7446	8.7029	8.6602	8.6385	8.6166	8.5944	8.572	8.5494	8.5264
4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.041	5.9988	5.9644	5.9117	5.8578	5.8025	5.7744	5.7459	5.717	5.6877	5.6581	5.6281
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351	4.6777	4.6188	4.5581	4.5272	4.4957	4.4638	4.4314	4.3985	4.365
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.099	4.06	3.9999	3.9381	3.8742	3.8415	3.8082	3.7743	3.7398	3.7047	3.6689
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.866	3.787	3.7257	3.6767	3.6365	3.5747	3.5107	3.4445	3.4105	3.3758	3.3404	3.3043	3.2674	3.2298
8	5.3177	4.459	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472	3.2839	3.2184	3.1503	3.1152	3.0794	3.0428	3.0053	2.9669	2.9276
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373	3.0729	3.0061	2.9365	2.9005	2.8637	2.8259	2.7872	2.7475	2.7067
10	4.9646	4.1028	3.7083	3.478	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782	2.913	2.845	2.774	2.7372	2.6996	2.6609	2.6211	2.5801	2.5379
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.948	2.8962	2.8536	2.7876	2.7186	2.6464	2.609	2.5705	2.5309	2.4901	2.448	2.4045
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534	2.6866	2.6169	2.5436	2.5055	2.4663	2.4259	2.3842	2.341	2.2962
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.671	2.6037	2.5331	2.4589	2.4202	2.3803	2.3392	2.2966	2.2524	2.2064
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022	2.5342	2.463	2.3879	2.3487	2.3082	2.2664	2.2229	2.1778	2.1307
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437	2.4753	2.4034	2.3275	2.2878	2.2468	2.2043	2.1601	2.1141	2.0658
16	4.494	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935	2.4247	2.3522	2.2756	2.2354	2.1938	2.1507	2.1058	2.0589	2.0096
17	4.4513	3.5915	3.1968	2.9647	2.81	2.6987	2.6143	2.548	2.4943	2.4499	2.3807	2.3077	2.2304	2.1898	2.1477	2.104	2.0584	2.0107	1.9604
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117	2.3421	2.2686	2.1906	2.1497	2.1071	2.0629	2.0166	1.9681	1.9168
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779	2.308	2.2341	2.1555	2.1141	2.0712	2.0264	1.9795	1.9302	1.878
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.599	2.514	2.4471	2.3928	2.3479	2.2776	2.2033	2.1242	2.0825	2.0391	1.9938	1.9464	1.8963	1.8432
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.366	2.321	2.2504	2.1757	2.096	2.054	2.0102	1.9645	1.9165	1.8657	1.8117
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419	2.2967	2.2258	2.1508	2.0707	2.0283	1.9842	1.938	1.8894	1.838	1.7831
23	4.2793	3.4221	3.028	2.7955	2.64	2.5277	2.4422	2.3748	2.3201	2.2747	2.2036	2.1282	2.0476	2.005	1.9605	1.9139	1.8648	1.8128	1.757
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002	2.2547	2.1834	2.1077	2.0267	1.9838	1.939	1.892	1.8424	1.7896	1.733
25	4.2417	3.3852	2.9912	2.7587	2.603	2.4904	2.4047	2.3371	2.2821	2.2365	2.1649	2.0889	2.0075	1.9643	1.9192	1.8718	1.8217	1.7684	1.711
26	4.2252	3.369	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655	2.2197	2.1479	2.0716	1.9898	1.9464	1.901	1.8533	1.8027	1.7488	1.6906
27	4.21	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501	2.2043	2.1323	2.0558	1.9736	1.9299	1.8842	1.8361	1.7851	1.7306	1.6717
28	4.196	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.236	2.19	2.1179	2.0411	1.9586	1.9147	1.8687	1.8203	1.7689	1.7138	1.6541
29	4.183	3.3277	2.934	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229	2.1768	2.1045	2.0275	1.9446	1.9005	1.8543	1.8055	1.7537	1.6981	1.6376
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107	2.1646	2.0921	2.0148	1.9317	1.8874	1.8409	1.7918	1.7396	1.6835	1.6223
40	4.0847	3.2317	2.8387	2.606	2.4495	2.3359	2.249	2.1802	2.124	2.0772	2.0035	1.9245	1.8389	1.7929	1.7444	1.6928	1.6373	1.5766	1.5089
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.097	2.0401	1.9926	1.9174	1.8364	1.748	1.7001	1.6491	1.5943	1.5343	1.4673	1.3893
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.175	2.0868	2.0164	1.9588	1.9105	1.8337	1.7505	1.6587	1.6084	1.5543	1.4952	1.429	1.3519	1.2539
Inf	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799	1.8307	1.7522	1.6664	1.5705	1.5173	1.4591	1.394	1.318	1.2214	1

What we'll need for the ANOVA

$$F = \frac{23.59}{0.76} = 31.04$$ WAY BIGGER THAN 3.6337!

☺

Well done!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